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Common Core
8th Grade Math
Volume II

by

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Introduction

This booklet employs a very linear approach to the Common Core Math 8 standards. Each standard is presented in the same order and language used in the New York State p-12 Common Core Learning Standards for Mathematics, which were published by the New York State Education Department. Interpretive paragraphs and examples are offered, along with links to instructional materials.

The Common Core Learning Standards are arranged into *Conceptual Categories*, *Domains*, *Clusters, Standards*, and sometimes *Standards components*. This is a hierarchical sorting system. You may remember the mnemonic from high-school: "Kings play chess on funny green squares"; with the "funny green squares" referring to the family, genus and species of a particular animal. In the Common Core learning standards, Domains, Clusters and Standards are your funny green squares, or -more directly- Domains are to Clusters and Standards as Family is to Genus and Species.

The *Appendix: Resources* section provides explanations of how to use resources from websites cited in this document, all of which are Open Source or Public Domain sites that do not require a subscription. Some of the textbooks cited are older, but 8th grade math and algebra has not changed significantly in the last 100 years, and carefully selected passages from older textbooks can still be very helpful. Where possible, links to a variety of instructional materials are offered to meet the learning needs of different students. It isn't necessary for a student to work through every link, but the student should keep working through the materials until they are confident they've mastered the standard concept.

Content

The order in which standards are addressed in this guide may seem slightly erratic. New York State advises that certain standards be addressed later in the school year, after students have been given an opportunity to master all preliminary material. All of the material addressed in this guide coincides with the selection of content outlined in the *State University of New York Educator's Guide to the Regents Examination in Math 8*. You will also notice some minor variations in how some of the standards are labeled when reviewing linked material. The systems used to label standards vary slightly by state, but the overall content structure remains constant. This guide covers the *Expressions and Equations and Functions* sections of the Math 8 standards.

Each section includes a *Conceptual Category* (shown centered and underlined in 14 pt type), a *Content Domain* (left-justified, Bold 14 pt type), the relative *Clusters* (12 pt bold text) and the *individual* *standards* (numbered alpha-numerically in the format “x-x.x”), some entries also include standard components, which are labeled with “a, b, c, etc...” Explanatory language and links to online resources follow the individual standards.

PDF copies of this study guide are available from the Baldwinsville Public Library website at:

http://www.bville.lib.ny.us/content/pdf_handouts/CC8mathV2.pdf

Writing a study guide like this will always involve some amount of trial and error. Although I have done my best to ensure accuracy, the reader may occasionally encounter small errors in the text, or be aware of other examples or materials which may be more effective. I am eager to hear from both math educators and students who have suggestions on how to improve this guide. Please direct your comments to the author's e-mail at: robertl@bville.lib.ny.us

Geometry

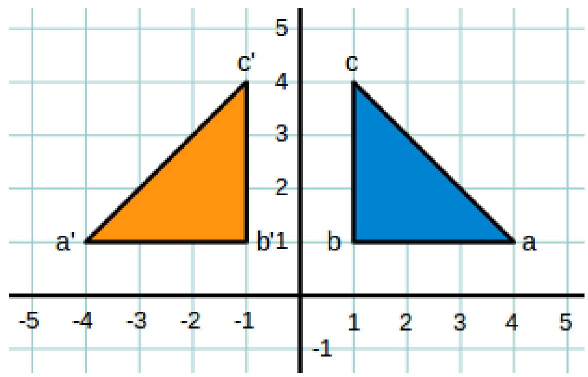
Understand congruence and similarity using physical models, transparencies, or geometry software.

8-G.1: Verify experimentally the properties of rotations, reflections, and translations:

Transformations are changes to the size, shape or location of a geometric figure. Rotations, reflections and translations are part of a special set of transformations that preserve congruency. Preserving the congruency of a figure means that the overall dimensions of that figure: including the lengths of sides and angle measures are all preserved.

8-G.1a: Lines are taken to lines, and line segments to line segments of the same length.

To prove that two lines are congruent, you must demonstrate that they have the same length. You can find the length of a line segment on the coordinate plane using the pythagorean theorem. In the example at the left, you see a triangle which has been reflected across the x-axis. To show that the sides of the original triangle (shown in blue) are comparable to the sides of the reflection (shown in orange) we check that $\overline{ab}$ is equal to $\overline{a'b'}$, that $\overline{bc}$ is equal to $\overline{b'c'}$ and that $\overline{ca}$ is equal to $\overline{c'a'}$.



- $\overline{ab}$ *is equal to* $\overline{a'b'}$: This first one is quite easy, because the point moves along the x-axis, but has no vertical movement, so we simply subtract the x-coordinate of point b from the x-coordinate of point a to get our line length.
  - $x_a - x_b$ implies $4 - 1 = 3$
  - $x_{a'} - x_{b'}$ implies $-4 - (-1) = -3$, since distances on the coordinate plane are measured in absolute values that -3 becomes $|-3| = 3$.
    - We've confirmed that the length of $\overline{ab}$ = the length of $\overline{a'b'}$ so that line is congruent
- $\overline{bc}$ is equal to $\overline{b'c'}$: This one is also easy, as the x coordinates for *a* and *b*, and *a'* and *b'* are the same, so we just have to subtract y-coordinate values.
  - $x_c - x_b$ implies $4 - 1 = 3$
  - $x_{c'} - x_{b'}$ implies $4 - 1 = 3$

- We've confirmed that the length of $\overline{bc}$ equals the length of $\overline{b'c'}$ so that line is also congruent

- $\overline{ca}$ is equal to $\overline{c'a'}$: For this you compare the coordinate using the equation:

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$$

For our example: $(1 - 4)^2 + (4 - 1)^2 = 18$ and $(-1 - \{-4\})^2 + (4 - 1)^2 = 18$

Which implie that c and c' both = $\sqrt{18}$ or $3\sqrt{2}$

Online resources that relate to this standard:

- **CK12.org: Segments and angles**
  http://www.ck12.org/user:YWtlZWxlckBhY2VsZnJlc25vLm9yZw../book/ACEL-Geometry-2012-2013/r2/section/1.4/
- **Khan Academy: Congruent line segments**
  https://www.khanacademy.org/math/basic-geo/basic-geo-lines/basic-geo-measuring-segments/v/congruent-segments

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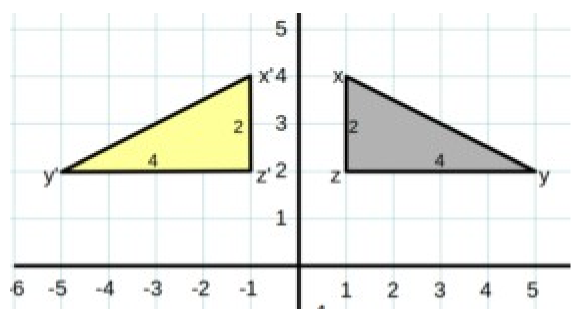
8-G1.b: Angles are taken to angles of the same measure.

When two angles are congruent it means they have the same measure. The simplest method for comparing angles in the coordinate plane is to find the tangent of the two angles you are comparing. The tangent of an angle is the ratio of the distance of the opposite side, versus the distance of the adjacent side. If two angles have the same tangent, then they have the same measure, and those two angles are congruent.



In the example at the right, the opposite side from ∠y is $\overline{zx}$, and the adjacent side is $\overline{zy}$. The measures of those lines are marked in the diagram as 2 and 4 units respectively. Therefore the tangent of ∠y and ∠y' is ½. Since the tangent of both angles are equal, we know that ∠s y and y' are equal, and therefore congruent.

Online resources related to this standard:

- **CK12.org: Congruent angles and angle bisectors**
  http://www.ck12.org/geometry/Congruent-Angles-and-Angle-Bisectors/
- **Khan Academy: Figuring out all the angles for congruent triangles**
  https://www.khanacademy.org/math/geometry/congruent-triangles/cong_triangle/v/figuring-out-all-the-angles-for-congruent-triangles-example

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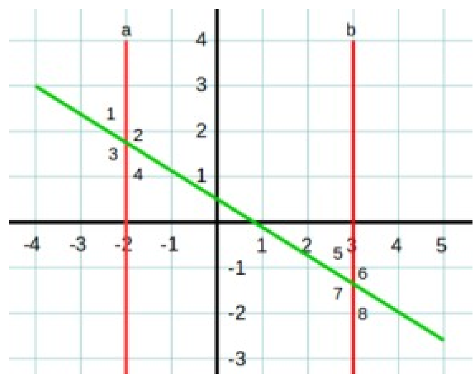
8-G1.c: Parallel lines are taken to parallel lines.



A transversal is a line that intersects two parallel lines. In the example at the right, the red lines marked a and b are parallel. The green line that intersects them is a transversal. So long as two lines are parallel, and the transversal that intersects them is a straight line, then you may assume that corresponding angles are of equal measure. That means that in this diagram ∠1 = ∠5, ∠2 = ∠6, ∠3 = ∠7, and ∠4 = ∠8. You can apply this principle to any situation in geometry where a straight line intersects a pair of parallel lines.

Online resources related to this standard:

- **CK12.org: Parallel lines and transversals**
  http://www.ck12.org/book/CK-12-Foundation-and-Leadership-Public-Schools-College-Access-Reader%3A-Geometry/section/2.6/
- **Khan Academy: Angles formed by parallel lines and transversals**
  https://www.khanacademy.org/math/basic-geo/basic-geo-angles/basic-geo-angle-relationships/v/figuring-out-angles-between-transversal-and-parallel-lines

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8-G.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Transformations are changes to figures in the coordinate plane. Rotations, reflections and translations are transformations that preserve congruency of figures. Two figures are congruent if they have identical size and shape. When you rotate, reflect or translate a figure, all side lengths and angles of that figure will remain the same, even though the figure has been moved to a different location in the coordinate plane.

Demonstrating congruence is a process of applying all of the methods described in standard 8-G.1 to show that the side-lengths and angle measures between two figures are equal. Congruent figures are those which have exactly the same size and shape.

Online resources related to this standard:

- **CK12.org: Congruent transformations**
  http://www.ck12.org/user:YWtlZWxlckBhY2VsZnJlc25vLm9yZw../book/ACEL-Geometry-2012-2013/r2/section/4.8/
- **Khan Academy: Transformations and congruence**
  https://www.khanacademy.org/math/geometry/congruent-triangles/transformations-congruence/v/testing-congruence-by-transformations-example

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8-G.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Reflections are achieved by multiplying the coordinates of a figure according to a particular pattern. The table below provides the forms for creating rotations along the x and y axes, and the diagonals $y = x$ and $y = -x$.

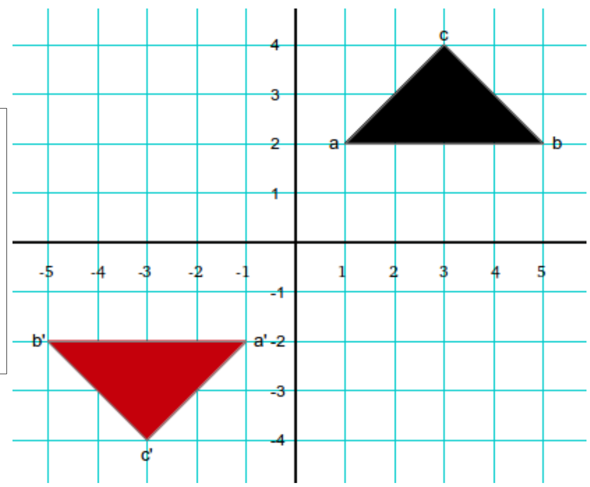
Reflection across the x-axis	$(x, y) > r_{x\text{-axis}}(-x, y)$
Reflection across the y-axis	$(x, y) > r_{y\text{-axis}}(x, -y)$
Reflection along the line y = x	$(x, y) > r_{y=x}(y, x)$
Reflection along the line y = -x	$(x, y) > r_{y=-x}(-y, -x)$

It's also possible to rotate figures by degrees in the coordinate plane. The table below demonstrates the pattern for rotations by intervals of 90°.

90° Rotation	$R_{90^\circ} = (x, y) = (-y, x)$
180° Rotation	$R_{180^\circ} = (x, y) = (-x, -y)$
270° Rotation	$R_{270^\circ} = (x, y) = (y, -x)$

In this example, a figure has been rotated 180º around the axis. If you look at the table above, you can see that the pattern for a 180º rotation is (x, y) > (-x, -y).
If we plug the original coordinates into a table, then apply the conversion described, we'll get the coordinates for the rotated figure.

(x, y)	(-x, -y)
a (1, 2)	a' (-1, -2)
b (5, 2)	b' (-5, -2)
c (3, 4)	c' (-3, -4)



Online resources related to this standard:

- **CK12.org: Translations, rotations and reflections**
  http://www.ck12.org/geometry/Translations-Rotations-and-Reflections/
- **Khan Academy: Geometry transformations**
  https://www.khanacademy.org/math/geometry/transformations

8-G.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Two figures that have the exact same shape and size are congruent. Two figures that have the exact same shape are called similar. Rotations, reflections and translations can all be used to move a figure across the coordinate plane while maintaining congruency. The word “dilate” means to get bigger, like the pupil of your eye does when you're in a dark room. In the coordinate plane, a dilation is a change in the size of a figure. If the size of a figure increases, but the measure of all its angles remain the same, then the original figure, and its enlarged copy are said to be similar. The term “dilation” is typically used to describe making a figure larger. When a figure is made smaller, it is said to be “scaled down”. All congruent figures are similar, but not all similar figures are congruent.

The first step to determining the process that led to the relocation or resizing of a figure is to create a table of values that includes the coordinates of the original figure, next to the coordinates of the transformed figure. This will help you to observe recurring patterns in the coordinates.

Triangle A	Triangle A'
X (1, 1)	X' (2, 2)
Y (3, 1)	Y' (6, 2)
Z (3, 3)	Z' (6, 6)

Notice how once you plug all of the figures into a table, it becomes immediately obvious that all of the coordinates were multiplied by 2, resulting in the figure doubling it's length and width, and moving up and to the right. Writing coordinates down in a table will often help to reveal patterns which may not be immediately obvious from just looking at your graph. Writing down coordinates in a table will also allow you to make use of the table of examples of transformations and rotations that was included in standard 8-G.3. Without bothering to graph the coordinates, look at the coordinates in the following table, compare those coordinates to the description of transformations in standard 8-G.3, and determine what kind of transformation is being represented.

Triangle m	Triangle n
Q (1, -3)	Q' (3, 1)
R (4, -3)	R' (3, 4)
S (4, 2)	S' (-2, 4)

After looking at the coordinates in the above table, and comparing the changes in those coordinates to the examples in standard 8-G.3, you should have noticed: $R_{90^\circ} = (x, y) = (-y, x)$. The pattern for manipulating coordinates to create a 90º rotation matches the pattern of changes made to the figures in our table, so we now know what kind of transformation this table represents.

Online resources related to this standard:

- **CK12.org: Dilation in the coordinate plane**
  http://www.ck12.org/geometry/Dilation-in-the-Coordinate-Plane/
- **Khan Academy: Dilations or scaling around a point**
  https://www.khanacademy.org/math/geometry/transformations/dilations-scaling/v/scaling-down-a-triangle-by-half

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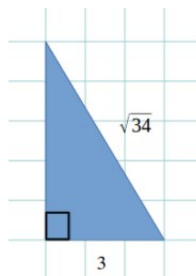
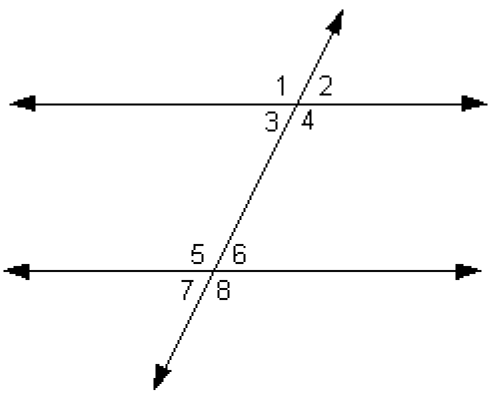
8-G.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

An informal argument is simply a plain language description of why a figure has certain traits. In the example at the right you can see that several traits of the triangle are labeled within the image:



- the box that's apparent at the bottom left is the symbol which indicates this is a right triangle, meaning that angle equals 90º.
- The bottom leg of the triangle is 3 units, and the length of the hypotenuse is $\sqrt{34}$

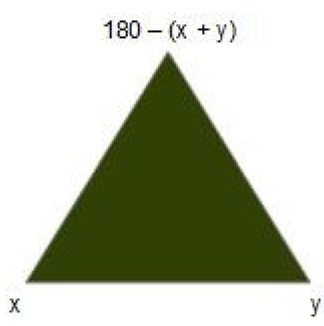
- $a^2+b^2=c^2$ so $3^2+b^2=34$ which means that $b^2 = 25$ so $b = 5$
- the tangent of the bottom right angle is 5/3, which indicates that angle is 59º
- Since the inside angles of a triangle add up to 180º, and the two lower angles equal 90º and 59º, that means the angle at the peak of the triangle equals 31º



When parallel lines are cut by a transversal, certain angles will be congruent, according to their orientation relative to the transversal. In the diagram at the left, all of the angles may be paired according to their relative positions:

- Angles 1, 4, 5 and 8 are all equal
- Angles 2, 3, 6 and 7 are all equal
-

The reason for this is that if the two lines are parallel then they have the same slope. Since they are intersected by a single straight line, the orientation of the two parallel lines relative to that intersecting line will be identical, therefore the angles that are created by the intersection of the lines will be congruent, so long as they are in the same position relative to the transversal. Another aspect of this is that any two adjacent angles have a sum of 180º, since angles 1 and 2 must equal 180º, and angles 1 and 3 must equal 180º, that must mean angles 2 and 3 are equal, because they both must equal 180º, minus the measure of angle 1. If you are given the measure of any single angle, you can calculate the measure of every angle created by the intersection of the transversal. So if you were looking at the diagram above, and you were told that angle 6 measured 70º, you could assume that angles 2, 3, 6 and 7 all equal 70º, and angles 1, 4, 5 and 8 all equal 110º.



Two triangles are similar if they have the same internal angles. Similarity is different from congruency, in that congruency implies the same angles and the same size, while similarity just means the triangles have the same angles. If you are given any two angles of a triangle, you can use that information to find the measure of the third angle, since the interior angles of a triangle will always have a sum of 180º. The illustration at the right shows how to find the measure of the third angle: you can call whichever two angles you're given your x and y values, and find the measure of the third angle by subtracting the sum of angles *x* and *y* from 180º. Likewise, if you know that two triangles both have a pair of identical angles, then you also know that those triangles are similar, because if any two of their angles are equal then the third must also be equal, according to the *180 – (x + y) property*.

Online resources related to this standard:

- **CK12.org: AA triangle similarity**
  http://www.ck12.org/book/CK-12-Geometry-Honors-Concepts/section/6.3/
- **CK12.org: Parallel lines and transversals**
  http://www.ck12.org/geometry/Parallel-Lines-and-Transversals/
- **Khan Academy: Triangle similarity**
  https://www.khanacademy.org/math/algebra-basics/core-algebra-geometry/copy-of-triangle_similarlity/e/similar_triangles_1
- **Khan Academy: Angles formed by parallel lines and transversals**
  https://www.khanacademy.org/math/basic-geo/basic-geo-angles/basic-geo-angle-relationships/v/angles-formed-by-parallel-lines-and-transversals

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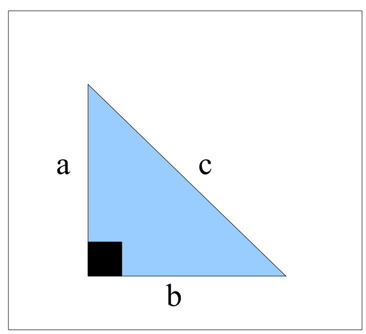
Understand and apply the Pythagorean Theorem

8-G.6: Explain a proof of the Pythagorean Theorem and its converse.



The Pythagorean theorem states that in right triangle, the length of $a^2 + b^2 = c^2$, where *a* and *b* are the two sides connected by the right angle, and c^2 is the square of the hypotenuse, which is the side of the triangle opposite the right angle. In the triangle shown at the right, the small square figure at the bottom left indicates this is a right triangle. There are several methods of proving the Pythagorean Theorem. This standard requires you to familiarize yourself with assorted proofs and and being able to explain why they are effective. The following website offers four proofs of the Pythagorean Theorem.

- http://www.opusmath.com/common-core-standards/8.g.6-explain-a-proof-of-the-pythagorean-theorem-and-its

Online resources related to this standard:

- **CK12.org: Pythagorean Theorem 1: proof and finding a missing side** http://www.ck12.org/book/CK-12-Foundation-and-Leadership-Public-Schools-College-Access-Reader%3A-Geometry/section/3.6/
- **Khan Academy: Pythagorean theorem proofs** https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-proofs/v/garfield-s-proof-of-the-pythagorean-theorem

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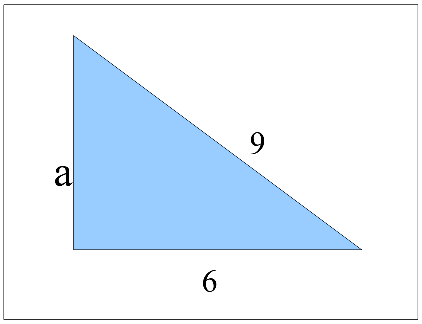
8-G.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.



The pythagorean theorem states that $a^2 + b^2 = c^2$, but a bit of simple algebra also shows us that:

- $c^2 - b^2 = a^2$ and $c^2 - a^2 = b^2$

In the example at the left the lengths of the bottom and hypotenuse of the triangle are labeled. From looking at the diagram, we see that: $a^2 + 36 = 81$. Those figures don't match the visual impression one gets from the diagram, and this is an example of why you should always go by the numbers that are printed alongside a diagram, and never try to estimate numbers by looking at the diagram. Figures that you encounter in geometry problems will often not be drawn to scale. Never assume that a figure is drawn to scale unless it's drawn on a coordinate plane, or marked as being drawn to scale. 81 – 36 = a2, which indicates that a^2 equals $\sqrt{45}$, which equals approximately 6.71. You can use this method anytime you need to find an unknown side length of a right triangle.

Online resources related to this standard:

- **CK12.org: Pythagorean Theorem 1: proof and finding a missing side**
  http://www.ck12.org/book/CK-12-Foundation-and-Leadership-Public-Schools-College-Access-Reader%3A-Geometry/section/3.6/
- **Khan Academy: The Pythagorean Theorem intro**
  https://www.khanacademy.org/math/geometry/right_triangles_topic/pyth_theor/v/the-pythagorean-theorem

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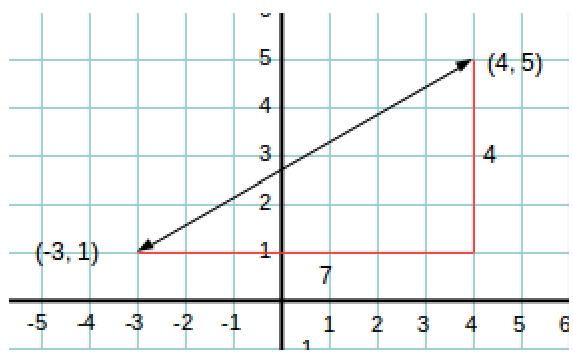
8-G.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

It's possible to find the distance between any two points on the coordinate plane by sketching two sides of a right triangle, and then using the Pythagorean Theorem to find the distance of the hypotenuse connecting those two points. In the example at the right, you can see two points at the end of a line: (-3, 1) and (4, 5). The two red lines are the two sides of the right triangle that connects these two points.



- In this example, sides we can refer to the side with a length of 4 units as *a*, and the side with a length of 7 units is side *b*.
- $16 + 49 = c^2$ or $c = \sqrt{16+49}$ or $\sqrt{65}$ or approximately 8.06 units

This method can be used to find the distance between any two points on the coordinate plane.

Online resources related to this standard:

- **CK12.org: Pythagorean Theorem to determine distance**
  http://www.ck12.org/trigonometry/Pythagorean-Theorem-to-Determine-Distance/
- **Khan Academy: Distances between points**
  https://www.khanacademy.org/math/geometry/analytic-geometry-topic/cc-distances-between-points/e/distance_formula

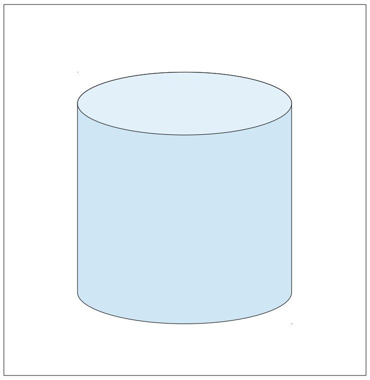
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

8.G.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

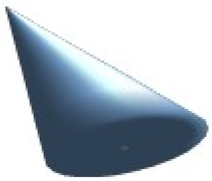
For this standard you first need to review the equation for finding the area of a circle, which is

$$A = \pi r^2$$

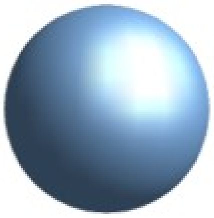
A refers to the area of the circle, π is a constant used in finding the area of a circle, and *r* refers to the radius, which is equal to ½ of the diameter of the circle.



The volume of a cylinder is found by $V = Ah$, where V is volume, A is the area of the circlular base of the cylinder, and h refers to the height of the cylinder. This equation may also be expressed with a "*b*" for "base" in place of the "*A*". It may also be expressed as $V = \pi r^2 h$. If you had a cylinder with a radius of 4", and a height of 6", you would get the equation V = π(16)(6) so you would have a volume of 96π in^3. If you were to use an approximation like 3.14 for π, you would have a volume of 301.44 in^3.



The volume of a cone is found by $V=\frac{1}{3}\pi r^2 h$, which is very similar to the equation for a cylinder except that you're dividing it by 1/3. If you had a cone that had a radius of 3cm and a height of 7cm, then your volume would equal $\frac{1}{3}\pi(3^2)(7)$ which would come out to $\frac{1}{3}(9)(7)\pi$ or 21π cm^3. Again, if we were using an approximation like π = 3.14, then our answer would be 65.94 cm^3.



The equation of the volume for a sphere is $V=\frac{4}{3}\pi r^2$. So, if you have a sphere with a volume of 2mm (about the size of a medium ball bearing) the volume of that bearing would be $\frac{4}{3}\pi 2^2$ or $\frac{16}{3}\pi$ if we once again use the approximation 3.14 for π, then we get $\frac{16}{3}(3.14)$ or roughly 16.75 mm^3.

Online resources related to this standard:

- **CK12.org: Surface area and volume of cylinders, cones and spheres**
  http://www.ck12.org/tebook/Foundation-and-Leadership-Public-Schools,-College-Access-Reader:-Geometry---Lesson-Plans-and-Exams/r1/section/8.0/
- **Khan Academy: Volume of cones, cylinders and spheres**
  https://www.khanacademy.org/math/basic-geo/basic-geo-volume-surface-area/basic-geo-volumes/e/volume-word-problems-with-cones--cylinders--and-spheres

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Number System

Know that there are numbers that are not rational, and approximate them by rational numbers

8-NS.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

A number is rational if it can be expressed as a ratio. All rational numbers have a terminating decimal value, or one that repeats continuously. A terminating decimal value is one such as 0.25 to express the value 1/4th. A continuously repeating decimal value is one such as $0.3\overline{3}$ to represent the value 1/3rd. The overline that you see over the second "3" in $0.3\overline{3}$ is to indicate that the number repeats continuously. Irrational numbers are those that repeat continuously, but which do not have a repeating pattern. Examples of irrational numbers include

π and $\sqrt{2}$.

Here is a list of irrational numbers and their decimal approximations
http://math.tutorcircle.com/number-sense/list-of-irrational-numbers.html

Aside from the number π, which is used to find the area of a circle, most of the irrational numbers you encounter will be the square roots of prime numbers. A prime number is one that cannot be reduced to any set of whole number multipliers besides 1 and itself. The number 40 is very clearly the product of 5 x 8, but no matter how much you try, you cannot find two integer values that you can multiply against each other to create the value 41. 41 is a prime number. The square roots of prime numbers are irrational.

Although you don't normally represent integers such as 1, 2, 3, etc. as fractions, they are considered rational numbers, because they may be expressed as values over 1: $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, etc. All fractions are considered rational numbers as a fraction is a ratio. Every rational number may also be expressed as a decimal value. You can find the decimal equivalent of a fraction by dividing the numerator (the number on top of your fraction) by the denominator (the number on the bottom of your fraction).

- The fraction $\frac{1}{8}$ can be reduced to a decimal value by dividing 1.00 by 8, to create the value 0.125. Because this value terminates in only 3 places, we can see that $\frac{1}{8}$ is clearly a rational number.
- You can also find the fractional value of a number by performing the opposite process. If you want to find out the fractional value of 0.03125, you enter into your calculator:

$$1 \div 0.03125$$

which gives you the value 32, since $\frac{1}{0.03125}$ = 32, we now know that 0.03125 = $\frac{1}{32}$.

Online resources related to this standard:

- **CK12.org: Rational and irrational numbers**
  http://www.ck12.org/section/Rational-and-Irrational-Numbers-%3A%3Aof%3A%3A-Using-Real-Numbers-and-Right-Triangles/
- **Khan Academy: Rational and irrational numbers**
  https://www.khanacademy.org/math/pre-algebra/order-of-operations/rational-irrational-numbers/v/introduction-to-rational-and-irrational-numbers

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8-NS.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

In high-school, the irrational number you'll encounter most often is π, which is equal to 3.141592...and continues on forever. Most of the time you'll approximate π as either 3.14 or 3.1416. The other irrational numbers you'll encounter will usually be roots of prime numbers. You can estimate the square roots of numbers algebraically. The most common method of approximating roots is to use a calculator. The online links in this section will introduce several additional methods for approximating square roots. The truly ambitious can learn to factor square roots manually, which is covered in page 146 of *Well's High School Algebra: topic 179*.

- https://archive.org/stream/newhighschoolalg00well#page/246/mode/2up

Online resources related to this standard:

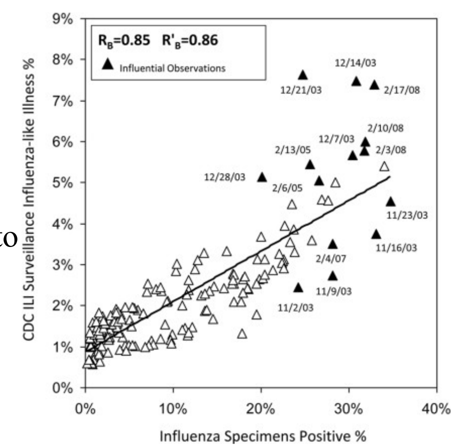
- **CK12.org: Approximate solutions to equations involving irrational numbers**
  http://www.ck12.org/algebra/Irrational-Square-Roots/lesson/Approximate-Solutions-to-Equations-Involving-Irrational-Numbers/
- **Khan Academy: Approximating square roots**
  https://www.khanacademy.org/math/pre-algebra/exponents-radicals/radical-radicals/v/approximating-square-roots

Statistics & Probability

Investigate patterns of association in bivariate data

8.SP.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

A scatter plot is a method for representing data with multiple quantitative variables. Remember that quantitative measures are employed to objectively measure information using some kind of standardized unit, while qualitative measures use subjective measures to gauge responses to or opinions of data.



http://openi.nlm.nih.gov/detailedresult.php?img=3083406_pone.0018687.g003&req=4

In the example at the left, you see a comparison of two methods of evaluating the prevalence of flu strains. In this example, the prevalence of influenza cases reported via the CDCs Virus Surveillance program was compared to the percentage of patients in a given region who saw their doctor to report flu-like symptoms, and ended up having a positive influenza test. In this graph, the *line of best fit* is already sketched onto the graph. There is a significant amount of clustering along that line. The statistical outliers in this data set are those points that are farthest away from the line of best fit. This graph represents a positive correlation, as an increase in one variable tends to correlate to an increase in the other. A negative correlation occurs when an increase in one variable leads to a decline in the other variable. This comparison shows a high degree of linear association, because the relationship between the variables creates a graph where the line of best fit is a straight line, rather than an arc or parabolic section. When a comparison like this one reveals a strong linear correlation between two types of data, and it's possitlbe to produce very similar results at multiple reporting sites, that data may eventually be used to create guidelines to help guide future expectations regarding the spread of influenza.

Online resources related to this standard:

- **CK12.org: Scatter plots and linear correlation**
  http://www.ck12.org/statistics/Scatter-Plots-and-Linear- Correlation/

- **Khan Academy: Constructing scatter plots**
  https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-data/cc-8th-scatterplots/e/constructing-scatter-plots

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8.SP.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Fitting a straight line means sketching a line that passes through your data points, and creates an impression of being the closest line possible to all of those data points. Your line of best fit must intersect as many points as possible, or at least appear to be a reasonable average of all the points in the graph. In later classes you may encounter the equations for formally fitting a line to data, but for now we'll focus on informal sketches, or on using web apps that can generate the equation of a line for you. The graph at the right shows a set of random data-points. The orange line shows the line of best fit for this graph. Notice how even though the line doesn't pass through any points, it does pass in between other points, and therefore represents a best fit for a data set where many of the x-coordinates show two values that are only separated by 1 unit on the y-axis.

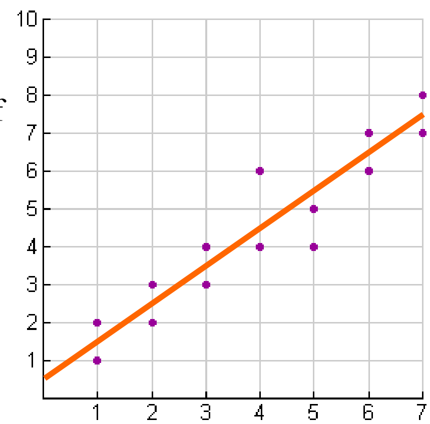

Image courtesy of regentsprep.org

You can use technology to find the equation of a line to fit a small data set, such as the *Illuminations Line of Best Fit* web-app: to use this app, enter your coordinate pairs at the bottom, next to the "Add Point" button, click "Add Point" to add your data point to the column. Once you have finished adding all of your coordinate pairs, check the *Show line of best fit* check-box, and you will see that line appear on the graph along with your data points, and you'll be given the equation of the line of best fit.

http://illuminations.nctm.org/Activity.aspx?id=4186

Online resources related to this standard:

- **CK12.org: Fitting lines to data**
  http://www.ck12.org/algebra/Fitting-Lines-to-Data/
- **Khan Academy: Fitting a line to data**
  https://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data

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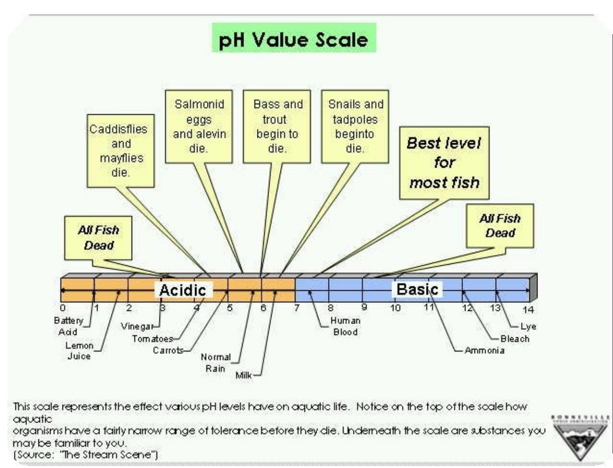
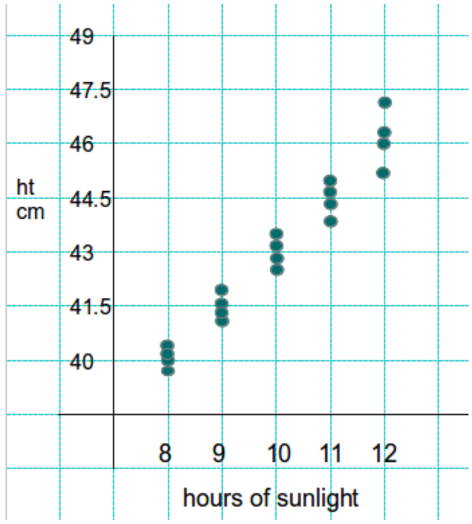
8-SP.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

When strong correlations are observed between multiple sets of data, and that correlation can be recreated by multiple researchers, then it becomes possible to use that data to guide expectations of those associations in future situations. You will see this phenomenon a great deal when you study chemistry. It's possible to predict the response that animals will have at different ph levels because those responses have been recorded and confirmed by numerous researchers. This standard asks you to be able to observe trends in data that you've gathered, so you can compare your data to standards, which are based on results that other researchers have gathered.



For the example given in the standard, imagine that you have chrysanthemums that are being grown in a seed company test lab; all of the plants are between 39 and 47.5 centimeters tall. A graph was created to show the average height of each plant, and the number of hours of sunlight those plants were exposed to while growing to maturity. In this example you can see that four plants were tested at each daily duration of sunlight. The number of hours of sunlight ranged from 8 to 12 hours. The height of the shortest plant was just under 40 centimeters, and the tallest plant was nearly 47.5 centimeters tall. It's also possible to observe that as the number of hours of sunlight per day increased, the degree of variation between the height of these plants increased, because the dots for 8 and 9 hours are bunched so close together it's difficult to distinguish between them, and the height of the plants that were exposed to twelve hours of sunlight appears to range from 45 cm to around 47 cm. Looking at this data set, we can say that these measurements show a strong linear correlation, because a line fitted to this data set would be straight. This is a positive correlation because an increase in one variable correlates to an increase in the other variable. This data set is also tightly clustered, and does not include any significant statistical outliers. When you look at a data set like this, you can use the information to estimate that if you put these plants in a location that received an average of 11 hours of direct sunlight per day, then (assuming proper moisture levels and soil acidity) you would probably have plants that were around 44.5 centimeters tall. However, when you are dealing with biology, you cannot assume that only 7 hours of sunlight per day would yield plants that were an average of 38.5 centimeters tall, or that 13 hours of sunlight per day would yield plants that were 47.5 cm in height. When dealing with living things you can only record the data you observe, and you should never make assumptions about growth habits based upon data within a given range.



Online resources related to this topic:

- **CK12.org: Fitting a line to data**
  http://www.ck12.org/book/CK-12-Algebra-I-Second-Edition/r1/section/5.3/

- **Khan Academy: Fitting a line to data** https://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data

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8-SP.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Bivariate data can be represented either on a graph, or on a two-way data table. The two way table shown below shows the results of a survey of preferred means of communication, divided by generational age cohorts. This data could be presented on a graph, but when you have data where individuals are divided into a relatively small number of categories, a two-way table can be a more useful means of presenting data. If you had done this same survey and asked for individual ages, and had those ages graphed along one of your axes rather than four discrete categories, you may choose to use a graph, but in this case the data-table offers a cleaner presentation of the data.

Preferred Means of Communicating

Generation	Cell Phone Call	Text Message	e-mail
Millenials	18%	64%	18%
Gen-X-ers	44%	8%	48%
Baby Busters	78%	7%	15%
Baby Boomers	84%	4%	12%

Based on the data in this table, a sociologist concludes that these results are not surprising, and that within each generational cohort, a majority of individuals seem to prefer those means of communicating that were popular when they were in their teens and twenties. Do you agree or disagree with this hypothesis? What other conclusions might you draw from this data set?

Based on the example named in the standard, imagine you survey a class of 28 students, and you get the following results:

	Curfew	No Curfew
Chores	18	2
No Chores	7	1

Based on these results, you would be able to confirm that students who have a curfew are somewhat more likely to have chores, but you cannot confirm the no curfew and no chores hypothesis, since there are so few students who are allowed to stay out as late as they'd like. 3 students out of 28 isn't a large enough sample to be able to draw any conclusion from the data. In order for information to be considered viable, it has to be "statistically significant", meaning there has to be a sufficiently large number of individuals within the sample group, or your results are regarded as being insignificant. You can make a hypothesis as long as you very clearly state that your conclusions only apply to the limited population that you sample, but you cannot suppose that your conclusion applies to all teens.

Online resources related to this sample:

- **CK12.org: Frequency tables to organize and display data**
  http://www.ck12.org/statistics/Frequency-Tables-to-Organize-and-Display-Data/
- **Khan Academy: Two-way tables**
  https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-data/two-way-tables/v/two-way-frequency-tables-and-venn-diagrams