BALDWINSVILLE
PUBLIC LIBRARY

# Common Core Algebra Booklet Volume II

by

Robert F. Loftus, MSLIS

Baldwinsville Public Library
Common Core Algebra Study Guide

# Introduction

This booklet employs a very linear approach to the Common Core Algebra standards. Each standard is presented in the same order and language used in the New York State p-12 Common Core Learning Standards for Mathematics, which were published by the New York State Education Department. Interpretive paragraphs and examples are offered, along with links to instructional materials.

The Common Core Learning Standards are arranged into *Conceptual Categories*, *Domains*, *Clusters, Standards*, and sometimes *Standards components*. This is a hierarchical sorting system. You may remember the mnemonic from high-school: “Kings play chess on funny green squares”; with the “funny green squares” referring to the family, genus and species of a particular animal. In the Common Core learning standards, Domains, Clusters and Standards are your funny green squares, or -more directly- Domains are to Clusters and Standards as Family is to Genus and Species.

The *Appendix: Resources* section provides explanations of how to use resources from websites cited in this document, all of which are Open Source or Public Domain sites that do not require a subscription. Some of the textbooks cited are older, but algebra -as taught at the high-school level- has not changed significantly in the last 100 years, and carefully selected passages from older textbooks can still be very helpful. Where possible, links to variety of instructional materials are offered to meet the learning needs of different students. It isn't necessary for a student to work through every link, but the student should keep working through the materials until they are confident they've mastered the standard concept.

## Content

This guide covers the *Functions*, *Statistics* and *Probability* domains. A second volume, covering the standards related to functions, statistics and probability is due to be released in January of 2015. You will notice that in some cases there are breaks in the numbering for particular content standards -such as when you see standard *F-BF.1 and F-BF.3, but no F-BF.2*. This is because the numbering system that applies to the Common Core Mathematics Standards originated with www.Corestandards.org, but the New York State Department of Education decided that students will not be tested on some of the standards, all of the material addressed in this guide coincides with the selection of content outlined in the *State University of New York Educator's Guide to the Regents Examination in Algebra I*. You will also notice some minor variations in how some of the standards are labeled when reviewing linked material. The suffixes used to label standards vary by state, but the overall content structure remains constant.

Each section includes a *Conceptual Category* (shown centered and underlined in 14 pt type), a *Content Domain* (left-justified, Bold 14 pt type), the relative *Clusters* (12 pt bold text) and the *individual standards* (numbered alpha-numerically in the format "x-x.x"), some entries also include standard components, which are labeled with "a, b, c, etc..." Explanatory language and links to online resources follow the individual standards.

PDF copies of this study guide are available from the Baldwinsville Public Library website at:

**http://www.bville.lib.ny.us/content/pdf_handouts/CCAB2014V1.pdf**

Writing a study guide like this is a process of trial and error. Although I have done my best to ensure accuracy, the reader may occasionally encounter small errors in the text, or be aware of other examples or materials which may be more effective. I am eager to hear from both math educators and students who have suggestions on how to improve this text. Please direct your comments to the author's e-mail at: robertl@bville.lib.ny.us

Thank You,

Robert F. Loftus, MSLIS

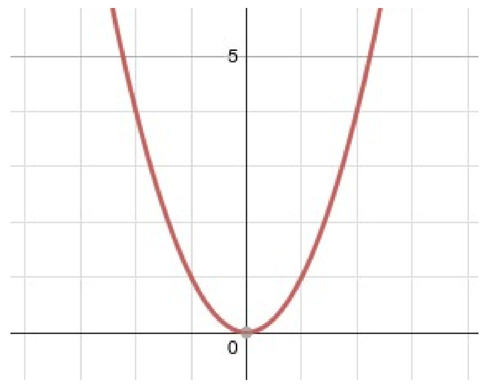
September 30th, 2014

# Interpreting Functions

## Understand the concept of a function and use function notation

**F-1F.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.**

A mathematical function is an equation where for each input value applied to the function, there is exactly one output value. Functions are represented using the format $y = f(x)$ (pronounced "f of x") and may also be represented as a graph, word problem or a table of values. One example of a function that's already familiar to you is the slope-intercept form of an equation: $y = mx + b$. In slope-intercept form, $x$ is your independent value, because it typically represents the range that you're selecting to graph; y is your dependent variable, because y varies according to which values of x that you choose to graph; $b$ is your y-intercept, which is the value of $y$ when $x = 0$. It is important to note that just because each value of x creates a single value of y as an output value, that does not mean that every value of y is unique. In the equation $y = x^2$, shown at the right, you can see that when $x = 1$ *or* $-1$, then $y = 1$; when $x = 2$ *or* $-2$, then $y = 4$. For each value of x there is a single value of y, but that does not mean that two separate x-values cannot have the same y-value.




$y = x^2$

Think of a pump with a rate of 600 gallons per hour, or ten gallons per minute. The amount of time the pump is switched on is your independent variable. The amount of water that the pump moves is your dependent variable, because the rate of flow is a constant, and the volume of water moved is ultimately a product of how long the pump is switched on. To represent this equation in the form $y = f(x)$, you would write $f(x) = 10x$, where $x$ is the time in minutes that the pump is switched on; if the pump runs for 1 minute, it pumps 10 gallons; if it runs for 3.2 minutes, it pumps 32 gallons; if it runs for 9 minutes, it pumps 90 gallons. If you were to graph this relationship, it would be a line with a slope of $m = 10$, and the domain of the function are all positive real numbers that could potentially represent an amount of time that the pump could be run.

**Online resources related to this standard**

- **CK12.org: Graphs and Functions**

  http://www.ck12.org/algebra/Graphs-and-Functions/

- **Khan Academy: Introduction to Functions**
  https://www.khanacademy.org/math/algebra/algebra-functions/classic-function-videos/v/introduction-to-functions

---

**F-1F.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.**

This standard expands upon the concept of *function notation*, which was introduced in the last standard. *Function notation* is a method of expressing mathematical relationships. When expressing an equation in slope-intercept form, you use the equation $y = mx + b$; function notation is very similar, except the $y$ is replaced by $f(x)$ (pronounced "f of x"). Function notation offers greater flexibility in mathematical terminology, and that flexibility will become more important as you progress in your study of mathematics. As mentioned in *F-1F.1*, a function is any mathematical relationship where each input value results in a single output value. Function problems often involve issues such as flow rates, worker productivity and rate of travel. Like any mathematical relationship, function problems may be expressed as an equation, a table of values, a graph or a written description. This standard deals with being able to convert table, graph or written data into equation form.

**Example:**

- The average output of a tile factory is 166.4 cases of tile per shift. A shift includes 4 workers, working for 8 hours. Calculate the average worker productivity per labor hour. A labor hour equals one employee working for one hour.
  - The problem specifically asks for how many cases of tile are being produced per labor hour. A shift includes four workers working for 8 hours, which means a shift contains 32 labor hours.
  - Productivity per labor hour would equal $\frac{[\textit{average productivity per shift}]}{[\textit{labor hours per shift}]}$ or $\frac{166.4}{32}$ which equals 5.2 cases per hour. If the average worker productivity is 5.2 cases per hour, and $x$ is our variable representing the number of labor hours worked, then $f(x) = 5.2x$.
  - The domain of this problem would be the range of possible values for the number of hours worked, or all possible real numbers that realistically represent a number of work hours the

factory might assign. If the factory does not permit overtime and only assigns workers up to 40 hours per week, and we know there are four workers then your domain for a typical week's production might equal all positive real numbers from 0 to 160.

- If the domain is 0 to 160 then the range would be all values along the line with the slope of *m=5.2*, and if f(x) = 5.2h, then the range for f(x) equals *5.2(0)* or *0* to *160(5.2)* or *832*.

Evaluating a function for inputs in its domain means to look at an output value, and use algebraic methods to work backwards and find out what the input value was. A construction estimator is assembling a bid for a patio job. He would like to submit a bid of less than $1400, to improve his chances of winning the job. If the company he works for has a policy that materials costs should never exceed 40% of the bid price, and the patio he's submitting a bid for is 8' x 8', then what is the limit of what he can spend for the 12” x 12” pavers that the job calls for? Estimate that the gravel fill for the job will cost $1.00 per square foot.

- The patio is 8' x 8', and the pavers are 12” x 12” -aka 1' x 1'- so we need 8 x 8 or 64 pavers.
  - We would also need 64 square feet, or $64 worth of gravel fill.
  - $1400 x 40% = $560
  - $560 - $64 = $496
  - $\$\frac{496}{64}$ = $7.75 per paver
  - *f(x) = $7.75p, where p = number of pavers*. The estimator now has his benchmark of how much the project will cost, and can calculate his other costs accordingly.

**Online resources related to this standard**

- **CK12.org: Algebraic Functions**
  http://www.ck12.org/algebra/Algebraic-Functions
- **Khan Academy: Domain and range of a function**
  https://www.khanacademy.org/math/algebra/algebra-functions/domain_and_range/v/domain-and-range-of-a-relation

**F-1F.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. Interpret functions that arise in applications in terms of the context.**

A mathematical sequence is a series of numbers where each value is generated by applying a common process of addition or multiplication is applied to the term or terms that precede each number in the sequence. Whereas in graphing a problem we often refer to an *x-value* to determine our domain, in a sequence we refer to an $a_n$ value. In a sequence problem, the value of $a_0$ is usually given, and the value of $a_1$ is found by applying the sequence function to the value of $a_0$. If "*f*"describes our mathematical operation then $a_1 = f(a_0)$, $a_2 = f(a_1)$, $a_3 = f(a_2)$, etc. An example of a sequence would be $a_n = a_{n-1} \times 0.5$. If $a_0$ equals 1024, then $a_1$ would equal 1024 x 0.5 or 512. If we continue this pattern we get $a_2 = 256$, $a_3 = 128$, $a_4 = 64$, $a_5 = 32$, etc.

The *Fibonacci sequence* is the numbers:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 35 etc.

The series begins with the numbers 0 and 1, and each subsequent value equals the sum of the two preceding values, or $a_n = a_{n-2} + a_{n-1}$. Comparing the simplicity of the *$a_n$ sequence description* to the recursive function description in the text of the standard demonstrates why this type of notation can be so useful. One example of a real world application of sequence problems is carbon dating, where the nuclear half-life of substances is measured according to a known deterioration period, to estimate the period of time since a material was last subjected to a significant chemical reaction.

**Online resources related to this standard**

- **Khan Academy: Radioactive decay: Half-life**
  https://www.khanacademy.org/science/chemistry/radioactive-decay/v/half-life
- **CK12.org: Arithmetic sequences**
  http://www.ck12.org/analysis/Arithmetic-Sequences/
- **Khan Academy: Arithmetic sequences**
  https://www.khanacademy.org/math/precalculus/seq_induction/seq_and_series/v/arithmetic-sequences

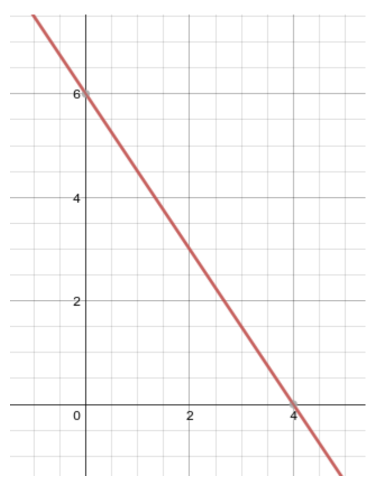
**F-1F.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

In your study of mathematics, you'll often see patterns that reoccur with certain types of problems. With practice you'll learn to recognize these patterns and make use of them to simplify problems. Examples of functions that model relationships and which follow specific patterns are equations for the area or volume of geometric figures. You know that the area of a square is $A=s^2$ , and the equation for graphing a circle with its center at the axis is $x^2+y^2=r^2$ .

- **Area of basic shapes**
  http://www.shmoop.com/basic-geometry/area-polygon-triangle-circle-square.html

You should also practice applying basic algebraic practices to convert equations to slope-intercept form *(y= mx + b)*. Being able to recognize common mathematical patterns such as those that arise in finding the area of geometric figures, and being able to manipulate equations to put them into slope-intercept form will improve your ability to handle the more complex equations that you'll encounter later in your mathematical studies.

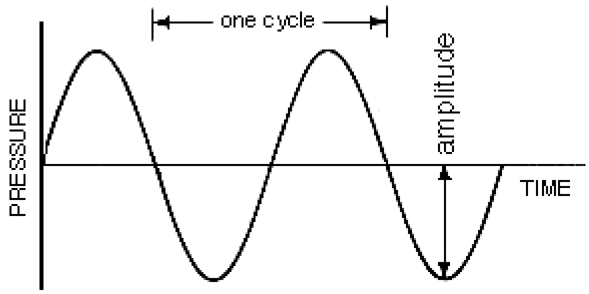
One method for finding the x and y intercepts of an equation is to put the equation in *ax + by = c* form and set the values of x and y to zero, then to solve for the remaining value. This will allow you to plot two points that you can connect to find the line that shows your solution set.




For example, look at the equation *3x +2y = 12*,

- When *x = 0* then *3(0) + 2y = 12*, or *2y = 12* which means *y = 6*. This means the first coordinate to mark on your graph is the point *(0, 6)*.
- When *y = 0* then *3x + 2(0) = 12*, or *3x = 12* which means *x = 4*. This means the second coordinate to mark on your graph will be *(4, 0)*.
  - Plotting the points where the line crosses the x and y axes, then connecting those two points

allows you to create a simple and quick sketch of the solution set for the equation.




Periodicity refers to an interval of one full cycle of a wave. In later mathematics studies and in physics you'll be asked to deal with problems involving sine waves. Graphs of sine waves are used to describe physical principles such as the flow of electric current, sound and light waves. The distance marked as "one cycle" in the graph at the left is the wavelength of this sine wave. The amplitude is the height of the wave. The frequency of a wave is measured in Hertz, and refers to cycles per second, so if you were graphing a wave with a frequency of 15 Hertz to a time period of one second, you would have 15 cycles within that one second interval.

**Online resources related to this standard:**

- **CK12.org: Graphs of Linear Equations**
  http://www.ck12.org/algebra/Graphs-of-Linear-Equations/lesson/Graphs-of-Linear-Equations/?referrer=featured_content
- **Khan Academy: Graphing Parabolas**
  https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
- **Math Open Reference: Cubic Function Explorer**
  http://www.mathopenref.com/cubicexplorer.html

---

**F-1F.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Relating the domain of a function to its graph means recognizing that you cannot always graph y values for every possible value of *x*. When dealing with problems drawn from real life, there are often practical limits to how you can represent a problem: if you're graphing the amount of product that a machine or a factory puts out per day, you aren't going to bother with graphing negative time values, or time intervals in excess of 24 hours; if you have a problem that asks you to graph the amount of liquid that a pump can move over a certain period of time, and that pump is only designed to run for an interval of up to 120 seconds, then you'll limit your graph to what the design of the pump can handle; if you're graphing the torque output of an electric traction motor as the voltage increases, you aren't going to graph output values for voltages that exceed a level which causes arcing, overheating and motor failure.

A fully loaded semi that's traveling on a flat highway at 55 mph while pulling a fully loaded 53' trailer consumes 11.8 gallons of diesel fuel per hour. At speeds above 55 mph, fuel consumption averages $11.8 + 0.03i^2$ gallons, where "i" is the increase in speed above 55 mph. Graph the rate of fuel consumption at 55, 60, 65 and 70 miles per hour.

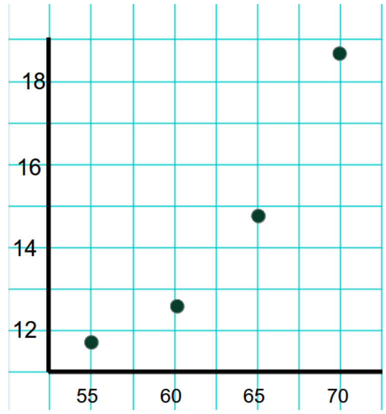
- This problem clearly states that you should graph values for a limited speed range. You are given the rate of fuel consumption at 55 mph, and asked to graph the rate of increase.
- Because we're given the speeds to graph, we're going to limit our x-values to just those that are named in the problem.
- Because 11.8 gallons per hour is the lower limit of our fuel consumption, we're going to graph from there up, and avoid leaving a large area of blank space at the bottom of our graph.

The first step in solving this problem is to create a table of values and plug in the parts of the equation.

| Speed | Rate of Increase | Fuel Consumption |
|---|---|---|
| 55 | 11.8 + 0.03(0) | 11.8 gph |
| 60 | 11.8 + 0.03(25) | 12.55 gph |
| 65 | 11.8 + 0.03(100) | 14.8 gph |
| 70 | 11.8 + 0.03(225) | 18.55 gph |

After creating our table of values, we create a graph of the problem. Because the problem asks us to graph the fuel consumption at specific points, we're using dots instead of a line, although a line could be fitted to these points to illustrate the general pattern of increasing fuel consumption as the truck speeds up.

The domain of this graph is 55, 60, 65 and 70 mph. The range associated with that domain are the values that we inserted into our table: 11.8 to 18.55 gallons per hour. We applied scales that are appropriate to the problem, to avoid leaving large areas of blank space in the graph. We also avoided graphing values that were not specified in the problem.



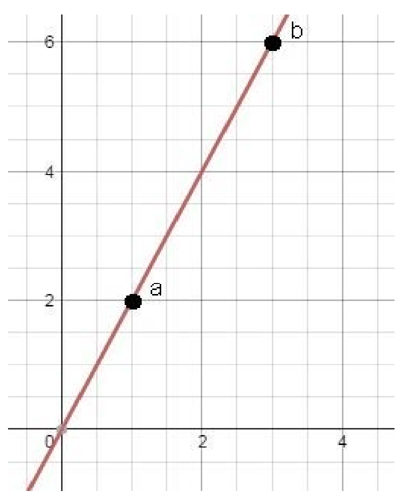

**Online resources related to this standard:**

- **CK12.org: Domain and range of a function**
  http://www.ck12.org/algebra/Domain-and-Range-of-a-Function/
- **Khan Academy: Domain and range of a function**
  https://www.khanacademy.org/math/algebra/algebra-functions/domain_and_range/v/domain-and-range-of-a-function-given-a-formula

---

**F-1F.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Analyze functions using different representations.**

The rate of change in a function is generally represented as "the change in your y-value over the change in your x-value". The Greek letter Delta – Δ – is used to represent change, so we can interpret rate of change as $\frac{\Delta y}{\Delta x}$. To find the rate of change of an equation over a particular interval, you use the equation $\frac{y_2 - y_1}{x_2 - x_1}$ where ($x_1$, $y_1$) are the coordinates of your starting point, and ($x_2$, $y_2$) are the coordinates of your end-point; the output of the equation will be the average slope of your graph over the interval you're observing.




**Example:** In the line graphed at the right, point a is at the location (1,

2) and point b is at location (3, 6). If we plug the values from these two points into our equation for finding the rate of change we get:

$$\frac{6-2}{3-1} \quad \text{or} \quad \frac{4}{2} \quad \text{or } m = 2$$

To simplify the finding of a slope, it's easiest to make the point to the right your ($x_2$, $y_2$) value, and make the point to the left your ($x_1$, $y_1$) value.

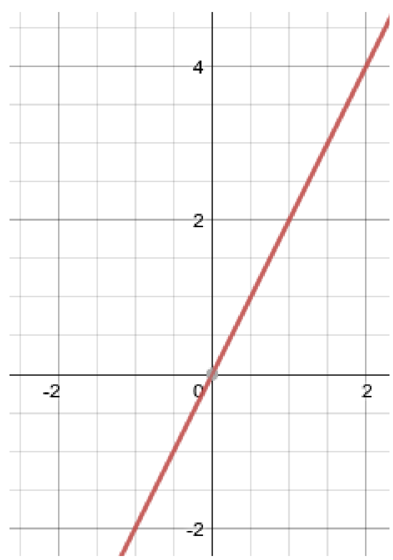
**Online resources related to this standard:**

- **CK12.org: Rates of change**
  http://www.ck12.org/algebra/Rates-of-Change/
- **Khan Academy: Slope of a line**
  https://www.khanacademy.org/math/algebra-basics/core-algebra-graphing-lines-slope/core-algebra-slope/v/slope-and-rate-of-change

---

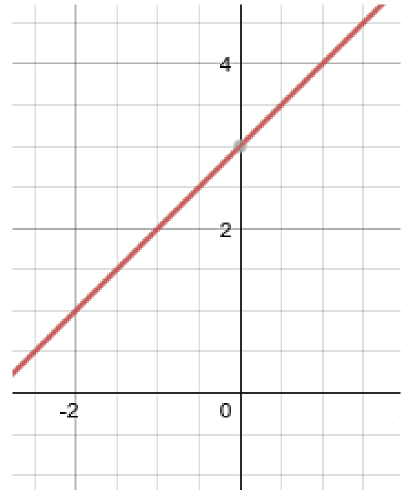
**F-1F.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.**

A linear equation is one where the graph of the equation is a straight line. You've already graphed linear equations in your earlier math classes. Linear equations may also be referred to as "equations in the first degree", as they do not include exponents greater than 1. The slope-intercept form of a linear equation is $y = ax+b$, where $a$ represents the slope of the line, and $b$ is the value of y where the graph crosses the y-axis. The form of a linear equation that passes through the origin is $y = ax$, because the y-intercept equals zero. In the example at the right $a = 2$. You create the graph of an equation by selecting a set of values of x that you will graph, creating a table of values, then plotting those values on your graph. Unless told otherwise, you will generally graph a range of values that include the origin of your graph. For the equation $y = x + 3$, we might begin by creating a table of values for the range of $x = -3$ to $x = +3$,



**Here is the graph of the equation y = 2x.**

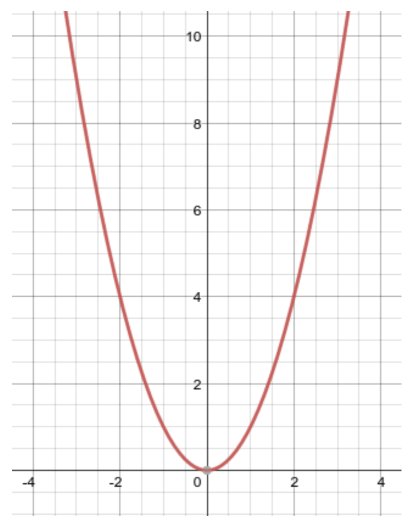
| x | x + 3 | y |
|---|---|---|
| -3 | -3 + 3 | 0 |
| -2 | -2 + 3 | 1 |
| -1 | -1 + 3 | 2 |
| 0 | 0 + 3 | 3 |
| 1 | 1 + 3 | 4 |
| 2 | 2 + 3 | 5 |
| 3 | 3 + 3 | 6 |




Plotting the points from the set of values we've calculated and drawing a line to connect them creates the graph shown at the right.

Graphing a quadratic function follows the same process, except the equation you're plotting will be of the second degree (meaning the exponent on the x-value equals 2). The standard form for a parabola with it's center at the origin is $y = x^2$. The process for calculating values of y and plotting your points on the graph is essentially the same. We'll graph $y = x^2$ values for $x = -3$ to $x = 3$ again.

| x | $y = x^2$ |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |




Once again, plotting out the points from the set of values we've created, and drawing a line to connect these points creates the graph you see at the left.

Finding the maxima or minima of a graph involves finding the maximum and minimum values of the equation within a given range of x-values. For a linear equation in slope intercept form: $y = ax + b$, when the value of $a$ is positive you know that the value of $y$ increases as the value of $x$ increases. If $a$ is negative then the value of $y$ decreases as $x$ increases. If you are asked to find the minima and maxima of an equation like $y = 3.2x$, when the domain of $x = 300$ *to* $500$, you look at the value of a, see that *3.2* is a positive number so the value will increase as x increases, then find your minimum by $3.2(300) = 960$ -which you know is the low end of your range because *3.2* is a positive number- and your maxima by $3.2(500) = 1600$. For a parabola whose vertex passes through the origin, the maxima or minima will be zero. If the parabola opens upward then zero is the minima, and if the parabola opens downward then zero is the maxima. Parabolas whose origins are not at the center are written in the form $y = a(x-h)^2 + k$, and the vertex of the parabola have an *x-value* equal to $h$ and a *y-value* equal to $k$. Again, if a is positive then the parabola will open upward and the *y-value* of your vertex will be a minima, and if a is negative then the parabola opens downward and the *y-value* of your vertex is a maxima.

**Online resources related to this standard:**

- **CK12.org: Graphs of linear equations**
  http://www.ck12.org/algebra/Graphs-of-Linear-Equations/
- **CK12.org: Graphing coordinates of linear functions**
  http://www.ck12.org/user:bHlhLnNuZWxsQGhlbnJ5LmsxMi5nYS51cw../book/CCGPS-Math-I-Coordinate-Algebra/section/3.6/
- **Khan Academy: Graphing parabolas**
  *https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/e/graphing_parabolas_0.5*

---

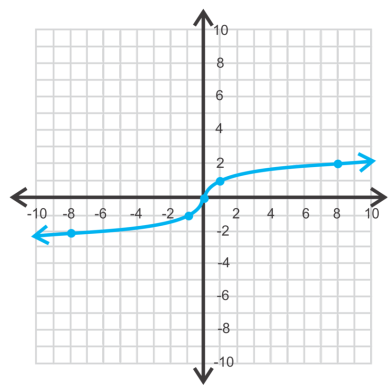
**F-1F.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.**

Before approaching these problems it is helpful to remember that roots of numbers can also be represented using fractional exponents; so $y=\sqrt{x}=x^{1/2}$ and the cube root of x can be represented as

$x^{1/3}$.

A square root function is one where your *y-value* increases according to the square root of *x*. A square root function has the slope intercept form of $y = \sqrt{x} + b$. You generally do not graph a square root function for x-values of less than zero, because the square root of a negative number would include the imaginary number *i*, which represents the square root of -1. You may be asked to graph these problems for values of positive and negative 7, since the square root of x may be positive or negative. To clarify: You may graph positive and negative values of *y* because *+3* and *-3* can both be square roots of *+9*, but you wouldn't graph this type of equation when *x = -9* because your roots would be *+3i* and *-3i*. Otherwise the process of selecting a range of x to graph, creating your table of values, then plotting your points and drawing a line to connect them is essentially identical to the process described in previous standards.

Graphing cube root problems is a bit different, because you will graph for negative values of *x*. Slope-intercept form for a cube-root equation is $y = x^{1/3} + b$. and have an appearance which is similar to the graph shown at the right. Once again, although the shape of the graph will be slightly different because you're graphing cube-root values, the process of selecting the range of x that you'll graph for, plugging those values into the slope-intercept form of your equation to create a table of values, then plotting the points and drawing a line to connect them is the same as graphing linear or square root problems.




$y = x^{1/3}$

Absolute value equations have the slope-intercept form $y = |ax + b|$. In an absolute value equation all of your values are positive, so when you graph for negative values of x, your y values will still be positive.

Piecewise functions are those which have a different slope intercept equation for different values of x. An example of a piecewise function might be:

- $y = 2x$ when $x < 10$

  $y = 1.8x$ when $x > 10$ but $< 20$

  $y = 1.6x$ when $x > 20$

Once again, you follow the same process of determining a range of x-values that you're going to graph, using the slope-intercept form of the equation to generate your

**Online resources related to this standard:**

- **CK12.org: Graphs of square root functions**
  http://www.ck12.org/algebra/Graphs-of-Square-Root-Functions/lesson/Graphing-Square-Root-Functions/?referrer=concept_details
- **CK12.org: Graphing cube root functions**
  http://www.ck12.org/analysis/Graphing-Cube-Root-Functions/
- **Khan Academy: Graphs of absolute value equations**
  https://www.khanacademy.org/math/algebra2/functions_and_graphs/piecewise-functions-tutorial/v/graphs-of-absolute-value-functions
- **Khan Academy.org: Graphing piecewise linear functions**
  https://www.khanacademy.org/math/algebra2/functions_and_graphs/piecewise-functions-tutorial/e/piecewise-graphs-linear

---

**F-1F.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.**

This is a rather broad standard with several components, many of which you've already addressed in your prior studies. We've already mentioned many times over in this study guide that there are four ways to express a mathematical relationship: a narrative description (aka: "a word problem"), an equation, a table of values, or a graph. A large part of your studies in high-school mathematics will consist of learning to translate information between these four forms.

This standard also includes being able to translate between the standard form and slope-intercept form of an equation, and knowing how to translate between numbers that include the square root symbol, and fractional exponents. Because this standard covers so much material, it's not practical to cover it all in this brief guide, so you should work your way through the material in the following linked sections.

**Online resources related to this standard:**

- **CK12.org: Forms of linear equations**
  http://www.ck12.org/algebra/Forms-of-Linear-Equations/

- **CK12.org: Fractional exponents**
  http://www.ck12.org/algebra/Fractional-Exponents/
- **Khan Academy: Converting to slope-intercept form**
  https://www.khanacademy.org/math/algebra-basics/core-algebra-graphing-lines-slope/core-algebra-graphing-slope-intercept/v/converting-to-slope-intercept-form
- **Khan Academy: Fractional exponents**
  https://www.khanacademy.org/math/algebra/exponent-equations/fractional-exponents-tut/e/exponents_3

---

**F-1F.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.**

Standard form for a quadratic function is $ax^2 + bx + c$. When graphed a quadratic function will create a parabola. Factoring a quadratic equation involves translating the standard form of the equation into a pair of multiples, which will reveal the location of the x-intercepts of the graph. An example of a

Before attempting to factor quadratic equations it's helpful to review the topic of multiplying binomials, particularly use of the *FOIL* method.

- https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/multiplying-binomials/v/multiplication-of-polynomials

With that review in hand, we can begin looking at ways of reversing that multiplication process, to convert a quadratic equation into a pair of binomials: Beginning with a quadratic equation in standard form:

- $x^2 - 5x - 6$, since the coefficient of $x^2 = 1$, then we know the equation must have the form $(x + a)(x + b)$ (remember it's appropriate to use addition signs here as subtraction may be defined as adding a negative value)
- since the coefficient of the final value = -6, we know that $a \times b = -6$
- *a + b must equal -5,* a bit of quick mental arithmetic suggests that our choices have integer value of either 1 and 6, or 2 and 3. -2 + -3 = -5, but -2 x -3 equals +6, not -6.
- +1 + (-6) = -5, and +1 x -6 = -6, so our solution set is probably (x + 1)(x - 6)

- Applying foil to (x + 1)(x - 6) gives us $x^2$ -6x + x -6, which reduces to $x^2$ -5x – 6.

Once you've found the values of your x-intercepts, you will select your range of x to graph. Your instructor will generally ask you to graph a range that is two or three units beyond the point of your x-intercepts.

**Online resources related to this standard:**

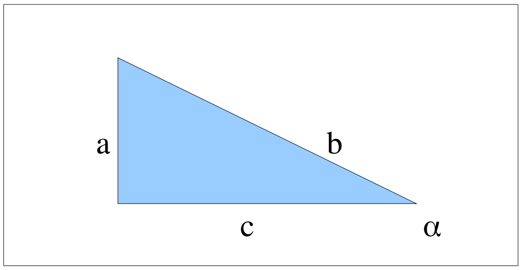
- **CK12.org: Factorization of quadratic expressions**
  http://www.ck12.org/algebra/Factorization-of-Quadratic-Expressions/
- **CK12.org: Completing the square**
  http://www.ck12.org/algebra/Completing-the-Square/
- **Khan Academy: Factoring quadratic expressions**
  https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/v/factoring-quadratic-expressions
- **Khan Academy: Solving quadratic equations by completing the square**
  https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/e/completing_the_square_1

---

**F-IF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.**

For this standard it's helpful to remember the algebraic cheer "Soh Cah Toa!" which is a mnemonic device for remembering the algebraic ratios for sin, cosine and tangent.

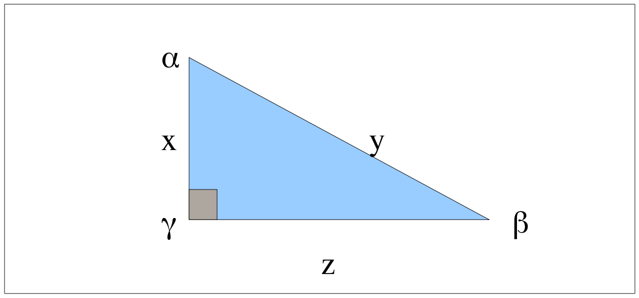
- $\sin = \frac{opposite}{hypotenuse}$
- $cosine = \frac{adjacent}{hypotenuse}$
- $tangent = \frac{opposite}{adjacent}$




These three values are referred to as "algebraic ratios" because they refer to the proportions of sides of a triangle. In the example shown here, the sin of ∠ α = $\frac{a}{b}$ , the cosine = $\frac{c}{b}$ and the tanget = $\frac{a}{c}$.

The addition and subtraction formulas for sin, cosine and tangent are:

- $\sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$




Don't be intimidated by the Greek letters. That symbol that looks like a lower-case a is the Greek symbol "alpha", and just represents a different way of representing a lower-case a. Greek symbols are frequently used in math and science calculations. The square at the base of the triangle indicates that angle is a right-angle, and therefore equals 90°.

- We'll begin by assigning the angle measures to create our example (note that this figure is not drawn to scale):
  - $\alpha = 60°$ $\beta = 30°$ $\gamma = 90°$
- We'll work through the formula for Sins. Since we know γ is a 90° angle, that means that (α + β) = 90° also.
- The sin of 90° = 1, so the equation becomes $1=(\sin\alpha)(\cos\beta)+(\cos\alpha)(\sin\beta)$
- If we assign the values we get: 1 = (sin 60°)(cos 30°) + (cos 60°)(sin 30°)
- Plugging in the values gives us: 1 = (0.866)(0.866) + (0.5)(0.5) = 0.7499 + 0.2500 = 1

**Online resources related to this standard:**

- **Khan Academy: Proof angle addition formulas**

  https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/angle-addition-formula-proofs/v/proof-angle-addition-sine

## Building Functions

### Build a function that models a relationship between two quantities.

**F-BF.1a: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.**

This standard relates to being able to look at a word problem and draw values for an equation from that description. An explicit expression is one that can be put into slope intercept form: $y = x + a$. A recursive process is one that generates a mathematical sequence, where each term's value is dependent upon applying a mathematical operation to the previous term in the sequence.

**Example:** A factory that produces food processors has two production lines. A finished unit comes off the end of line A and is ready for packaging once every minute. Line B was constructed later and has more modern components, and a finished unit comes off the end of line B and is ready for packing every 50 seconds. How many food processors will come off the line if it runs for 1 hour.

- Because line A producers 1 unit per minute the equation is $y = m$, where $m$ is the number of minutes that line A runs for. Line B works faster, and only has a 50 second interval between units coming off the line, which translates into $y=\frac{6}{5}m$ . If we add those two values together, we get $y=\frac{5}{5}m+\frac{6}{5}m$ or $y=\frac{11}{5}m$ .
- To find the number of units that come off the line in one hour: $y=\frac{11\,x\,60\,minutes}{5}$ or $y=\frac{660}{5}$ or 132 units per hour.

Now that we know how many units are produced per hour, you can find out the number of units produced during an 8 hour shift, or in a month with 23 working days.

Recursive processes are those that involve creating a mathematical sequence where a term in the sequence is found by applying a mathematical operation to the prior term in the sequence. Recursive processes are often applied to problems related to growth or decay.

**Example:** A tree farm that raises trees to be pulped for newspapers uses a particular strain of white pine whose above-ground wood mass increases at an average rate of 80% per year. If the seedlings that are planted at the farm have an average above ground wood mass of 3 pounds, create a table of values showing how much above ground wood mass the trees will have for the first five years after they are planted. (round to the nearest $10^{th}$).

| Year | Mass x 1.8 | Wood pulp mass in lbs. |
|---|---|---|
| 0 | 3 | 3 |
| 1 | 3(1.8) | 5.4 |
| 2 | 5.4(1.8) | 9.7 |
| 3 | 9.7(1.8) | 17.5 |
| 4 | 17.5(1.8) | 31.5 |
| 5 | 31.5(1.8) | 56.7 |

**Online resources related to this standard:**

- **CK12.org: Fitting lines to data**
  http://www.ck12.org/algebra/Fitting-Lines-to-Data/lesson/user:bHlhLnNuZWxsQGhlbnJ5LmsxMi5nYS51cw../Build-a-function-that-models-a-relationship-between-two-quantities.MCC9-12.F.BF.1-1a-1b-2/
- **Khan Academy: Word Problems**
  https://www.khanacademy.org/search?page_search_query=word+problems

---

**F-BF.3: Identify the effect on the graph of replacing f(x) by *f(x) + k*, k f(x), *f(kx)*, and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.**

Adding a constant to a function can change the value of a function, by adding, subtracting, mulitiplying or dividing a set of results by a common factor, the shape of a graph can be changed dramatically.

*f(kx)* will multiply all of your y-values by k. Remember that the figure *f(x)* is very similar to the slope-intercept form of $y = ax + b$. Therefore, the figure *f(kx)* is equivalent to $y = akx + b$; the value of $k$ becomes a multiplier that increases the slope of the line you're graphing.

$f(x) + k$ will increase all of your y-values by the amount of *k, moving the graph vertically.* This translation may also be thought of as $y = ax + b + k$. Remember that $k$ can be a positive or negative value, causing the figure you are graphing to move up or down the y-axis.

$f(x + k)$ will shift a curve horizontally. The best way to illustrate this concept may be to study graphs of parabolas that are given in vertex form, so you can see immediately how the addition of the $k$ value can shift the graph. Work through the Graphing Quadratic Equations study guide, paying particular attention to the second page which deals with parabolas that do not have their vertex at the origin.

Online resources related to this standard:

- **CK12.org: Graphing quadratic equations study guide**
  http://www.ck12.org/algebra/Quadratic-Functions-and-Their-Graphs/studyguide/Graphing-Quadratic-Equations-Study-Guide/?referrer=concept_details
- Khan Academy: Graphing parabolas in vertex form
  https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/e/graphing_parabolas_1

---

## Linear, Quadratic and Exponential Models

### Construct and compare linear, quadratic, and exponential models and solve problems.

**F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.**

First degree equations are represented with linear functions. A first degree equation is one that when put into slope-intercept form of $y = ax + b$, the exponent of $x = 1$. Common examples of linear equations include the time to work and cost functions that were described earlier in this handout. A first degree is referred to as a linear function because when the equation is graphed, the figure that you graph is a straight line. Linear functions are those where a common unit, which is determined by the slope of the line is added over a regular interval.

Exponential functions are those that when put into slope-intercept form have an exponent for x that is greater than 1. The equation $y = x^2$ is an example of an exponential equation. Examples of exponential graphs include graphs of area, volume and compound interest problems.

**Online resources related to this standard:**

- **CK12.org: Linear, exponential and quadratic models**
  http://www.ck12.org/algebra/Linear-Exponential-and-Quadratic-Models/lesson/user:Y2NvcmJsZXlAZndwcy5vcmc./Linear-Exponential-and-Quadratic-Models-F.LE.3/?referrer=concept_details
- **Khan Academy:Understanding linear and exponential models**
  https://www.khanacademy.org/math/algebra/algebra-functions/one-variable-modeling/e/understanding-linear-and-exponential-models

---

**F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**

With earlier standards you were asked to find the slope of an equation using two points. Once you've found the slope of a line it's possible to find the y-intercept of the line by taking any point along that line, and using the equation $y - mx$, where y is the y-coordinate of your point, x is the value of your x-coordinate, and m is the slope of the line. For example, if you have a line that includes the points (1,4) and (3,5), it's possible to realize that the rise of your equation is 1 over a run of 2. Since you know your slope is ½, you can now calculate your x-intercept by 4 – 1 (½) to get 3.5, so you know the equation of your line is y = ½x + 3.5, or f(x) = ½x + 3.5.

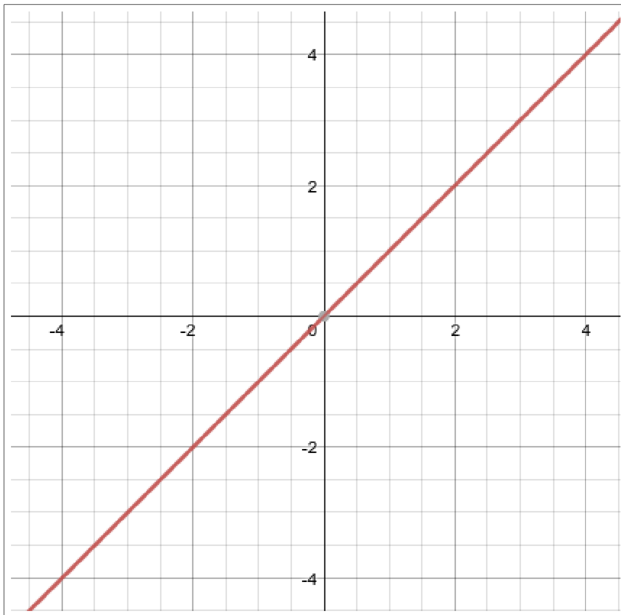
**Online resources related to this standard (review of content from F-LE.1)**

- **CK12.org: Linear, exponential and quadratic models**
  http://www.ck12.org/algebra/Linear-Exponential-and-Quadratic-Models/lesson/user:Y2NvcmJsZXlAZndwcy5vcmc./Linear-Exponential-and-Quadratic-Models-F.LE.3/?referrer=concept_details
- **Khan Academy:Understanding linear and exponential models**
  https://www.khanacademy.org/math/algebra/algebra-functions/one-variable-modeling/e/understanding-linear-and-exponential-models

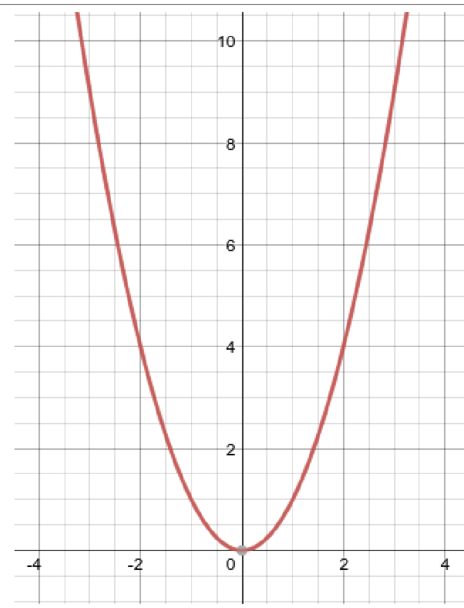
---

**F-LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.**

This standard is quite simple, and really only requires two graphs to observe the difference.




$y = x$




$y = x^2$

As you can see in this example, the y-value of the second graph begins increasing much more rapidly as the absolute value of x increases.  A simple table makes this immediately apparent.

**y = x** **$y = x^2$**

| x | y | x | y |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 4 |
| 3 | 3 | 3 | 9 |

**Online resources related to this standard (review of content from F-LE.1)**

- **CK12.org: Linear, exponential and quadratic models**
  http://www.ck12.org/algebra/Linear-Exponential-and-Quadratic-Models/lesson/user:Y2NvcmJsZXlAZndwcy5vcmc./Linear-Exponential-and-Quadratic-Models-F.LE.3/?referrer=concept_details
- **Khan Academy:Understanding linear and exponential models**
  https://www.khanacademy.org/math/algebra/algebra-functions/one-variable-modeling/e/understanding-linear-and-exponential-models

---

**F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.**

Once again we'll repeat how a mathematical relationship may be presented in any one of four ways: as an equation, a table of values, a graph, or a narrative description (commonly referred to as a "word problem"). This standard is asking you to recognize the elements of an equation from a narrative description.

**Example:** Ralph pays a flat delivery fee of $17.95 per month, plus 11.5 cents per kilowatt hour for electricity. When he goes down to the laundry room of his apartment he is able to see his electric meter. When Ralph looks at his old electric bill he saw that last month his meter ended at 74288 on April 28th, and his next reading is today – May 29th. Ralph goes downstairs and sees that his electric meter is at 74491. Approximately how much will Ralph have to pay on his next electric bill? Assume that what the meter-reader saw was roughly the same as what Ralph saw, and use 74491 as your basis for estimating.

- First we look at the two numbers Ralph has: 74491 – 74288 = 203. Since it's common knowledge that electric meters in the US give values in Kilowatt-Hours, we know that means Ralph used 203 kilowatt hours of electricity during the month.

- Since Ralph pays $17,95 for a flat delivery fee, plus 11.5 cents per kilowatt-hour, that means his bill will be $17.95 + 203(.115) = 41.295, which we round up to $41.30.
- Ralphs small electric bill suggests that he lives in a very small apartment, or the weather was very mild that month.

**Online resources related to this standard:**

- **CK12.org: Graphs and functions**
  http://www.ck12.org/algebra/Graphs-and-Functions/
- **Khan Academy: Functions and their graphs**
  https://www.khanacademy.org/math/algebra2/functions_and_graphs

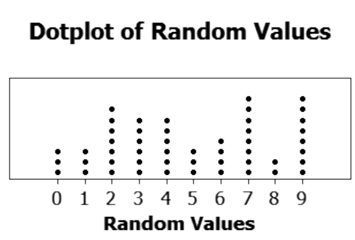
---

# Statistics and Probability

## Interpreting Categorical & Quantitative Data

**Summarize, represent, and interpret data on a single count or measurement variable**

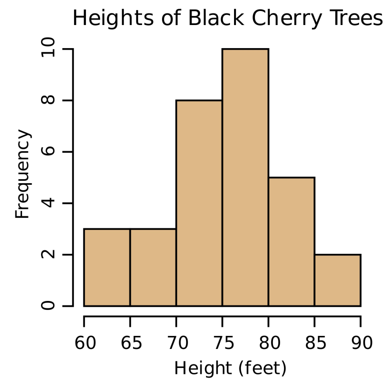
**S-ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots)**.




**Courtesy of Wikipedia.org**

Dot plots are used for showing a distribution of values when the values represented are whole units. Since the graph at the left is random, we can plug in any imaginary values we'd like for this example. Imagine that the numbers along the bottom represent a quick poll of the number of writing instruments -pens or pencils- that students have in their backpack. In this example there are 3 students who have no pens or pencils in their backpack, 3 students who have only one pen or pencil in their backpack, 7 who have 2 pens or pencils in their backpack, etc. Dot plots are a simple way to represent data distributions, where the data represented is composed of whole units (we aren't counting a broken pen as half a writing utensil).

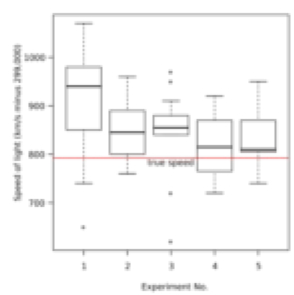
A histogram is essentially the same thing as a dot-plot, except you use solid bars with a number line along the side of the graph, rather than individual dots to show your distribution of results. The example at the left shows a distribution of a range of heights for a group of black cherry trees in a given area. Histograms have all the same potential applications for representing data as dot-plots, but are considered to have a more attractive appearance, and to be easier to read. One way to think of the relationship between histograms and dot-plots is to think of dot-plots as your short-hand method for gathering data, and a histogram is how you will present that same data when you assemble a report.




**Histogram image courtesy of wikipedia.org**

Box plots are used when you want to represent data that has a range of possible values.

The boxes in the graph typically represent the middle 50% of your data distribution, with the dividing line in the box showing the mean -or average= of that 50%. The "whiskers" that extend above and below the box show the distribution of high and low quartiles of your data set. Box plots are referred to as a "quartile" representation of of data because each box or whisker represents ¼ of your result set.



**Online resources related to this standard:**

- **CK12.org: Box plots and histograms**
  http://www.ck12.org/book/CK-12-Texas-Instruments-Algebra-I-Student-Edition/r1/section/12.3/
- **Khan Academy: Creating histograms**
  https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/creating-histograms/e/creating-histograms
- **Khan Academy: Box and whisker plots**
  https://www.khanacademy.org/math/probability/descriptive-statistics/box-and-whisker-plots/v/reading-box-and-whisker-plots

**S-ID.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.**

This standard ultimately comes down to using your statistical data honestly and effectively. When presenting a summary of statistical information, you should always endeavor to present that information in a way that provides the fullest possible picture of your data set. Unlike the media, where pundits frequently cite headline numbers and ignore any countervailing facts or adjustments to an information set, math students are expected to be more honest and thorough in their presentation of data.

As you learn about different types of graphs, think about how those graphs may be used to represent characteristics of a data set: understand that using a simple line graph to represent a data set that is highly variable, with many statistical outliers is inherently dishonest; if your evaluation produces data with a wide range of results that you need to make an effort to illustrate the range of those results; provide correlation data when possible, so that people may not mistake a widely disbursed data set for solid consistent results, or assume that very tight, consistent results are just a general observation.

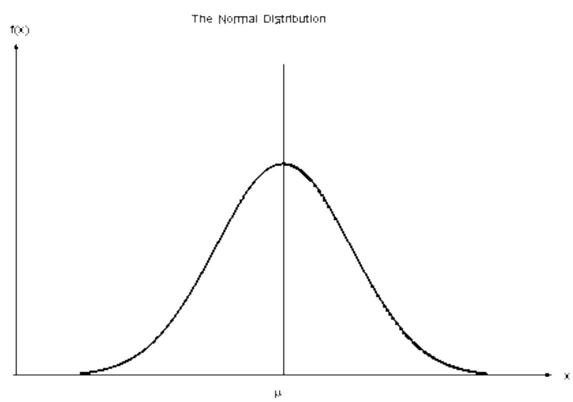
The interquartile range of a data-set separates your data into a middle 50%, and a top and bottom 25% of outlying results. Look back at standard S-ID.1, and the description of box-plots for an illustration of this concept. Standard of deviation refers to the difference between the mean -or average- of all your results, and the majority of your statistical data. The smaller your standard of deviation, the more consistent and tightly-grouped your data set.

**Online resources related to this standard:**

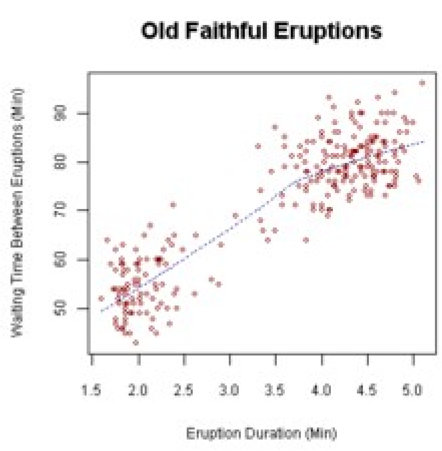
- **CK12.org: Applications of variance and standard deviation**
  http://www.ck12.org/statistics/Applications-of-Variance-and-Standard-Deviation/
- **Khan Academy: Calculating the mean interquartile range**
  https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6-mad-and-iqr/e/calculating-the-interquartile-range--iqr-

**S-ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).**

Students should be able to look at a set of data points and realize trends in the data. Results may be skewed to the right or left of a graph. A "normal" distribution will be a symmetrical mound of data that occupies the center of the graph. Statistical outliers are results that are significantly different from the other results and which occupy the extreme ends of your data distribution. Students should also be able to observe trends in the data that occur over time intervals. Students should also adjust their data presentation so as to present a normal appearance. While it is academically dishonest to manipulate results to show your desired result, it is simply good form to adjust the scale, domain and range of your graph so as to present a "normal" data distribution.




**This image represents a "normal" data distribution. Image courtesy of http://www.itl.nist.gov/**




Students should also be able to observe trends that are apparent in a data set. In the example at the left you see a graph of the eruptions at the Old Faithful geyser. This graph compares the time between eruptions, to the duration of each eruption. There is a clearly visible trend which shows that the duration of an eruption is related to the amount of time between that eruption and the previous eruption – quite simply; the longer the geyser sits brewing between eruptions, the longer the eruptions that follow that brewing period will be. Students should be capable of picking out these kinds of trends in a data set.

**Online resources related to this standard:**

- **CK12.org: Descriptive statistics**
  http://www.ck12.org/user:MCPS/book/Unit-3:-Descriptive-Statistics/r6/section/1.0/

- **Khan Academy: Measures of central tendency**
  https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/statistics-intro-mean-median-and-mode

---

**S-ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.**

Mean and standard deviation are two methods for identifying patterns and trends of a data set. The shmoop website provides an excellent tutorial on this topic. *Mean* is another way of referring to the average value of a data set. Unfortunately, because you can get a wide range of values within a data-set, a simple average of all your results rarely gives a sufficient picture of the value of the data. Finding the standard of deviation can provide a more complete picture of your results. The shmoop website provides an excellent tutorial on this topic.

- **Shmoop.com: Standard deviations**
  http://www.shmoop.com/common-core-standards/ccss-hs-s-id-4.html

Using the Mean of a data-set is acceptable when you have results that are very consistent, and all of your data points are tightly bunched on your graph. When your data shows a wide degree of variation, then using the Mean of your results can create a false impression of data consistency, so calculating Standard of Deviation is a more appropriate measure to use. In some fields, such as medical studies, research protocols dictate that all studies include the sharing of results and calculating the Standard of Deviation for your reuslts.

**Online resources related to this standard:**

- **CK12.org: Standard deviation of a data set**
  http://www.ck12.org/statistics/Standard-Deviation-of-a-Data-Set/
- **Khan Academy: Variance and standard deviations**
  https://www.khanacademy.org/math/probability/descriptive-statistics/variance_std_deviation/v/exploring-standard-deviation-1-module

---

**S-ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.**

Categorical data refers to information that's used to separate portions of a tatistical sample into groups. Examples include categories such as age, race, gender, level of educational achievement, etc. A two way frequency table breaks data down into only two categories. This information can help to provide a better picture of the behavior of groups that are represented by your data.

**Example:** Between 2009 and 2013, tests were given to rate nationwide to determine students preparedness to pursue STEM (Science, Technology, Engineering and Mathematics) studies. The year that appears in parentheses indicates the year that students were tested.

| % who scored as “proficient” | Science | Math |
|---|---|---|
| 4th graders | 34% (2009) | 42% (2013) |
| 8th graders | 32% (2011) | 35% (2013) |

Source: https://www.nms.org/AboutNMSI/TheSTEMCrisis/STEMEducationStatistics.aspx

Naturally, using this kind of table for your own data requires you to think about how to arrange the information so it can be presented in this format. Look at the Source link for this data, and look at how the information was extrapolated from the article and put into the table. Notice how on the website the data is presented in a series of bullet points, and in our table here that data is arranged into the table so that it's readily scannable. The addition of the dates provides context regarding when testing of students took place.

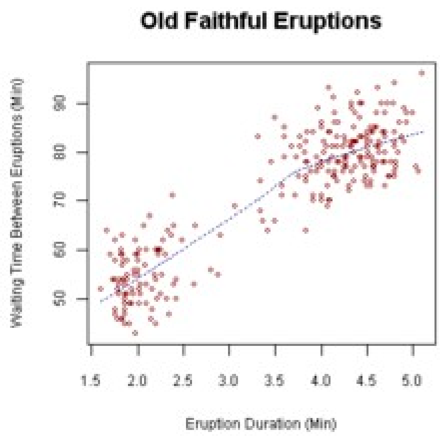
**Online resources related to this standard:**

- **CK12.org: Two way frequency tables**
  http://www.ck12.org/book/CK-12-Geometry-Honors-Concepts/section/11.4/
- Khan Academy: Two way-frequency tables
  https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-data/two-way-tables/e/two-way-frequency-tables

**S-ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.**

A scatter plot is a method for representing datawith multiple variables. In the example at the left, you can see how the bottom axis shows the eruption duration, and the left axis shows the time between eruptions. These are quantitative measures because they both represent measures employing standard units. Remember that quantitative measures are employed to objectively measure information using some kind of standardized unit, while qualitative measures use subjective measures to gauge responses or opinions to data.

Scatter plots can be used to show correlations that occur over a period of time. In the example you see here, the pattern of the data shows that the longer the period of time between eruptions, the greater the duration of those eruptions.




**Online resources related to this standard:**

- **CK12.org: Scatter plots and linear correlation**
  http://www.ck12.org/statistics/Scatter-Plots-and-Linear-Correlation/
- Khan Academy: Constructing scatter plots
  https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-data/cc-8th-scatter-plots/e/constructing-scatter-plots

---

**S-ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.**

This standard requires you to be able to look at the information in a problem, and to be able to identify your independent and dependent variables. Generally speaking, you will always manage a problem so that the information that's given to you in the problem is your independent variable, and the information that you're trying to calculate is your dependent variable. That means if you're given a time and distance problem that states the time spent in travel and asks you to find the rate of speed, then the travel time is your independent value -your x-value range- and your y-value is your rate of speed. If you're given an amount of travel time and distance and asked to find the speed, then the speed is your dependent variable.

**Example:** A train travels 148 miles to get from the Regional Transportation Center in Syracuse, New York to the train station in Albany. How fast will the train have to travel to make the trip in only 2 hours, in 2.5 hours, in 3 hours? In this problem we're given the distance and assorted travel times and asked to find how fast the train would have to travel. Since the distance is a constant that doesn't change regardless of the speed or travel time we aren't going to represent that on our graph. We'll graph the travel times of 2, 2.5 and 3 hours along the x-axis, and make the speed in mph our y-values.

**Example:** More complex examples of this issue arise when you get into studies of statistics, sociology and economics. The Bureau of Labor Statistics publishes a variety of different measures of unemployment. The “Headline Rate” of unemployment that's often discussed in the media is referred to as the “U3 Unemployment Rate”. There is a relationship in the economy between the U3 rate and wage growth for American workers. Historically, wage growth begins to accelerate when the U3 rate falls below 5.5%, and decelerates when the U3 unemployment rate exceeds 5.5%. This is because when U3 is high there are more skilled workers who are seeking jobs, and when U3 is low most of those workers are employed, meaning employers have to hire less skilled workers, or those with less desirable work histories, or pay more to get the most desirable workers. Because the unemployment rate is a measure of the number of people who are seeking work, and variation in wage rates occurs because of the availability, or lack of desirable job-seekers, a graph relating the U3 unemployment rate and wage growth would feature the U3 rate as your independent variable, and wage growth as your dependent variable.

**Online resources related to this standard:**

- **CK12.org: Interpret linear models**
  http://www.ck12.org/user:bHlhLnNuZWxsQGhlbnJ5LmsxMi5nYS51cw../book/CCGPS-Math-I-Coordinate-Algebra/r273/section/4.5/
- **Khan Academy: Linear models of bivariate data**
  https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-data/cc-8th-patterns-in-data/e/linear-models-of-bivariate-data

**S-ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.**

The method of manually calculating a correlation coefficient is beyond what you will do in most high-school algebra classes, but it is helpful to know what a correlation coefficient is. When looking at statistical data, you will often see an *"r"* value. The closer your r-value is to $\pm 1$, the more consistent your data-set is, meaning the more compact and tightly bunched all of your data points will appear on your graph. If the slope of the line you've fitted to the graph is positive then you'll have an r-value approaching $+1$, and when that slope is negative you'll have an r-value approaching $-1$. When a data set is loosely grouped, with a variety of near random results, a variety of statistical outliers that are significantly different from your other results and little in the way of any sense of organization, then you will have an r-value that is close to 0.

If you have data and you would like to calculate an r-value for that data set, you can use this free online calculator which is recommended by the CK12.org website:

https://www.easycalculation.com/statistics/correlation.php

**Online resources related to this standard:**

- **CK12.org: Linear correlation coefficient**
  http://www.ck12.org/book/CK-12-Probability-and-Statistics-Concepts/section/11.2/
- **Khan Academy: Linear regression and correlation**
  https://www.khanacademy.org/math/probability/regression/regression-correlation/v/correlation-and-causality

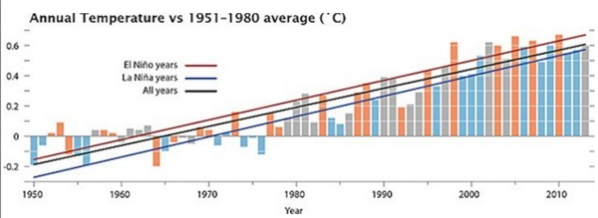
---

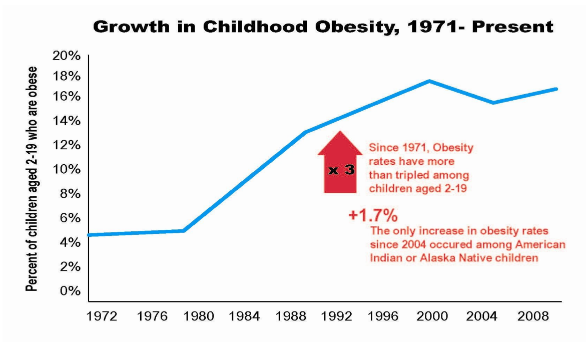
**S-ID.9: Distinguish between correlation and causation.**

Correlation occurs when two values appear to vary in kind with one another. Causation means that one value varies as a result of changes in another value. Just because two values show some coincidental change in value, does not automatically imply the two values are correlated. To show causation, there must be a supporting argument, which is a rational explanation as to why the observed correlation exists. A supporting argument may include reference to the results of a controlled experiment, references to commonly accepted scientific principles or laws, or the results of prior observations that share a significant degree of similarity.

**Because events can correlate coincidentally, simple correlation does not prove causation. If there is no compelling argument to support the existence of causation between data sets, then the assumption of correlation is considered to be false.**

A confounding variable is a third factor that is not referred to in the original data, but which is known, or is likely to be a causal factor related to changes in your data that you've observed. Willfully ignoring confounding variables can lead to ridiculous fallacies based upon your observations. One example of a ridiculous fallacy is the claim that "Global warming is caused by obese children." If we look at the data, we see two charts that appear to correlate with one another:




If we were trying to be ridiculous, we could state that the correlation between these two data sets clearly prove that global warming is caused by childhood obesity, but because we are rational people, we know that such an assertion is clearly nonsense. You can see examples of statistical fallacies at the *Spurious Correlations* website http://www.tylervigen.com. Although we use ridiculous examples to demonstrate the topic of correlation versus causation, the real threat of misuse of information has to do with arguments that seem plausible at first, but which -upon further examination- turn out to be false. This is a topic you'll address in greater detail in your later mathematical studies, and in social sciences classes, where statistical data is often used as a means of trying to predict and evaluate human behavior.

**Example: The Fallacy Game:** The Fallacy Game is a way to have fun with data by ignoring confounding variables - and a lesson that sometimes the best way to learn something is to intentionally do it wrong. Go online and find examples of data sets that appear to correlate with one another, and come up with the most ridiculous explanation you can to falsely claim the two data sets are related.

**Online resources related to this standard:**

- **CK12.org: Correlation and causation**
  http://www.ck12.org/earth-science/Correlation-and-Causation/
- **Khan Academy: Correlation and causality**
  https://www.khanacademy.org/math/probability/statistical-studies/types-of-studies/v/correlation-and-causality