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Common Core Algebra Booklet

Volume I

by

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Introduction

This booklet employs a very linear approach to the Common Core Algebra standards. Each standard is presented in the same order and language used in the New York State p-12 Common Core Learning Standards for Mathematics, which were published by the New York State Education Department. Interpretive paragraphs and examples are offered, along with links to instructional materials.

The Common Core Learning Standards are arranged into *Conceptual Categories*, *Domains*, *Clusters, Standards*, and sometimes *Standards components*. This is a hierarchical sorting system. You may remember the mnemonic from high-school: “Kings play chess on funny green squares”; with the “funny green squares” referring to the family, genus and species of a particular animal. In the Common Core learning standards, Domains, Clusters and Standards are your funny green squares, or -more directly- Domains are to Clusters and Standards as Family is to Genus and Species.

The *Appendix: Resources* section provides explanations of how to use resources from websites cited in this document, all of which are Open Source or Public Domain sites that do not require a subscription. Some of the textbooks cited are older, but algebra -as taught at the high-school level- has not changed significantly in the last 100 years, and carefully selected passages from older textbooks can still be very helpful. Where possible, links to variety of instructional materials are offered to meet the learning needs of different students. It isn't necessary for a student to work through every link, but the student should keep working through the materials until they are confident they've mastered the standard concept.

Content

This guide covers the *Number and Quantity* and *Algebra* domains. A second volume, covering the standards related to functions, statistics and probability is due to be released in January of 2015. You will notice that in some cases there are breaks in the numbering for particular content standards -such as when you see standard *A-APR.1* and *A-APR.3*, but no *A-APR.2*. This is because the numbering system that applies to the Common Core Mathematics Standards originated with www.Corestandards.org, but the New York State Department of Education decided that students will not be tested on some of the standards, all of the material addressed in this guide coincides with the selection of content outlined in the *State University of New York Educator's Guide to the Regents Examination in Algebra I*. You will also notice some minor variations in how some of the standards are labeled when reviewing linked material. The suffixes used to label standards vary by state, but the overall content structure remains constant.

Each section includes a *Conceptual Category* (shown centered and underlined in 14 pt type), a *Content*

Domain (left-justified, Bold 14 pt type), the relative *Clusters* (12 pt bold text) and the *individual standards* (numbered alpha-numerically in the format “x-x.x”), some entries also include standard components, which are labeled with “a, b, c, etc...” Explanatory language and links to online resources follow the individual standards.

PDF copies of this study guide are available from the Baldwinsville Public Library website at:

http://www.bville.lib.ny.us/content/pdf_handouts/CCAB2014V1.pdf

Writing a study guide like this will always involve some amount of trial and error. Although I have done my best to ensure accuracy, the reader may occasionally encounter small errors in the text, or be aware of other examples or materials which may be more effective. I am eager to hear from both math educators and students who have suggestions on how to improve this text. Please direct your comments to the author's e-mail at: robertl@bville.lib.ny.us

Thank You,
Robert F. Loftus, MSLIS
September 30th, 2014

Number and Quantity

Reason quantitatively and use units to solve problems.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

This standard includes:

- Knowing the names and symbols used to represent common units of measure and identifying the units used in a problem
- Knowing common conversion values such as 12 inches per foot, 5280 feet per mile, 1 cubic centimeter per milliliter, and 1000 milliliters per liter
- Recognizing when unit conversions must be applied and using conversion factors and tables
- Recognizing units used on a graph or scale drawing, and relating unit scale to actual dimensions
- Being able to recognize and use non-standard units such as boxes of tile, gallons of paint, gross of bolts, etc.

A unit is a quantity of stuff that we use as a benchmark for measuring other quantities of stuff. The stuff being measured may be time, distance, volume, or mass. Some units represent rates, which is a measure of one quantity of stuff against another, such as miles per hour: where you are comparing distance traveled over a period of time. Sometimes you will have *non-standard* units, such as when you're trying to estimate how many gallons of paint you'll need to finish a job. If a gallon of paint covers 300 square feet of wall space, then you divide the area of the space being painted by 300 square feet to get the number of buckets of paint you should buy.

The ultimate goal of studying the use of units is to prepare the student for developing skills of Quantification, which is the ability to understand evaluation metrics and statistical data which are commonly used in business and the social sciences, such as Net sales per square foot of retail space or understanding the difference between total population statistics versus per capita statistics. Quantification also relates to the ability to use equations from the sciences such as rate = distance/time or Ohm's Law.

Online resources related to standard N-Q.1:

- CK12.org: Measurement
  http://www.ck12.org/measurement: Go to the site and review each of the modules listed below
- Khan Academy: Number and Quantity
  https://www.khanacademy.org/commoncore/grade-HSN-N-Q

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling. Reason quantitatively and use units to solve problems.

In mathematics the term "Descriptive Modeling" refers to selecting appropriate terms and figures for the problem at hand. In the N-Q.1 standard, it was mentioned how in some problems non-standard units may be used. Construction estimating provides numerous examples of problems where an individual has to deal with non-standard units. When calculating the quantity and cost of materials needed for a construction job, the contractor often has to deal with finding the area covered by a tile or paver, spacing between units, coverage amounts, etc.

Reggie has a contract to replace the mop tubs and quarry tile floors in the janitor's closets of an office building. There are 5 janitor's closets. Two closets are 4' x 6', while the other 3 are 3' x 5'. The mop tubs in the three smaller closets are 36" square. The mop tubs in the larger closets are 30" x 48". All of the mop tubs are built into the floor, so the area of the tubs should be subtracted from the floor area when calculating how much tile to purchase. The tiles the building owner has specified cost $48.88 for a box of 36, and are 6" square. How much will it cost Reggie to buy the tile for this job?

- Sometimes it's helpful with these kinds of problems to take a piece of scrap paper and make simple sketches to aid in picturing the job.

One method of approaching this problem is to count up the total number of tiles, and compare it to the number of tiles per box.

- Calculate the amount of space to be tiled. The smaller closets are 3' by 5', and contain a 36" square mop tub; we subtract an area of 3' x 3' from each space, which means the area being tiled in each of those closets is only 2' x 3'.
- Remember the tiles are 6" square, 2' x 3' = 24" x 36", so the area is found by calculating

  $\frac{24}{6} inches\ x \frac{36}{6}$ or *4 tiles x 6 tiles = 24 tiles per closet*.
- *24 tiles per closet x 3 closets = 72 tiles* to do the smaller closets.
- There are 36 tiles per box, 72 divided by 36 means Reggie needs two boxes of tile to do the 3 smaller closets.

Another approach involves looking at the amount of square footage per closet, and dividing by the coverage per box of tile.

- Since the tiles are 6" square, we know it takes four tiles to fill one square foot. 36 tiles per box, divided by 4 tiles per sf of coverage, means that each box of tile will cover an area of 9 sf.
- The larger closets are 4' x 6', and have mop tubs that are 30" x 48". That's 24 sf per closet, with the tubs

taking up an area of 2.5' x 4', or 10 sf per closet. This means Reggie has to tile 14 sf of area per large closet, or 28 sf total. The boxes of tile only cover 9 sf each, so Reggie has to buy 4 boxes of tile, because the problem does not mention the option of buying a partial box, or loose tiles.

- When we add the two boxes of tile for the smaller closets to the four boxes of tile for the larger closets, we see that Reggie needs six boxes of tile for this job. Since the building owner specified a particular type of tile that costs $48.88 per box, we find the cost by multiplying *6 boxes x $48.88 per box*, and see that Reggie's tile cost for the job will be $293.28.
- CK12.org: Measurement
  http://www.ck12.org/measurement: Go to the site and review each of the modules listed below
- Khan Academy: Number and Quantity
  https://www.khanacademy.org/commoncore/grade-HSN-N-Q

N-Q.3 Choose a level of accuracy appropriate to the limitations on measurement when reporting quantities.

This standard refers to the use of *Significant Figures.* “Significant Figures” are defined as numbers that carry meaning for a value. This includes:

- All non-zero values, for example: 2.97 has three significant figures. All three figures contribute to the expression of value.
- Zeros that are bracketed by non-zero numbers, for example: 32.03 has four significant figures, because the zero is bracketed by non-zero values, it is included as a significant figure.
- Leading zeros are not considered Significant Figures, for example: if you saw a digital display that read “000032.5”, you would eliminate the four leading zeros, and only record the value “32.5” as significant figures.
- When multiplying two numbers that include decimal values, the answer you give should have the same number of significant figures following the decimal point as the component value with the least significant figures. For example, if you were multiplying 32.3 x 27.928, your calculator will produce an answer of 902.0744, but because the first number has only one digit following the decimal point, you would round your answer to 902.1, because your component value with the least significant figures (in this case that is 32.3) only has one number to the right of the decimal point.

Online Resources related to this standard

- CK12.org: Significant Figures

http://www.ck12.org/chemistry/Significant-Figures/

- Khan Academy: Significant Figures

  https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/sig-figs-pre-alg/v/significant-figures

The Real Number System

Use Properties of rational and irrational numbers

N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A number is referred to as "rational", if it can be written as a fraction. A fraction is a form of ratio. A number is irrational if it cannot be expressed as a fraction. A rational number can always be written as either a ratio of two integers or a decimal value. When written as a decimal value, a rational number will always terminate, or have a continuous repeating value.

- $\frac{1}{4}$ is a rational number, because it is an expression representing a relationship between two integer values. When expressed as a decimal, the value equals 0.25, which terminates after two places.
- $\frac{1}{3}$ is also a rational number, because it represents a relationship between two integer values, and although the decimal value does not terminate, it has a continuous repeating value of $0.33\overline{3}$.
- Integer values such as 2 or 3 are considered rational, as they are equivalent to $\frac{2}{1}$ or $\frac{3}{1}$.
- $\frac{\pi}{3}$ is not a rational number, because π is not an integer, and has no terminating decimal value.
- The sum or product of two rational numbers will always be rational, because you are adding or multiplying values that may be expressed as ratios of integer values.
- If a, b, c and d are all integers, then $\frac{a}{b} x \frac{c}{d}$ equals $\frac{ac}{bd}$
- Or, using an example with integers: $\frac{2}{3} x \frac{3}{4}$ equals $\frac{6}{12}$ which equals $\frac{1}{2}$
- Decimal values can often be expressed as simplified relationships of integer values if you multiply by a power of 10. $\frac{1.76}{4}$ x $10^2 = \frac{176}{400} \div \frac{4}{4} = \frac{44}{100}$ *remember that dividing by a value such as* $\frac{4}{4}$ *is*

equivalent to dividing by one, and is a method for reducing fractions to their lowest common denominator. We see it's possible to reduce the value again so: $\frac{44}{100} \div \frac{4}{4} = \frac{11}{25}$

- If a = π, b=3, c=3 and d=4, then your answer will never be a rational number, because the value of π will always be part of your equation: $\frac{\pi}{3} x \frac{3}{4} = \frac{(3\pi)}{12}$ *which equals* $\frac{\pi}{4}$ *the irrational number remains*.
- In some cases you will substitute a decimal or fractional value for an irrational number, in the case of π, you may use the fraction $\frac{22}{7}$ or the decimal value 3.1416, which creates a slight change in the value of your answer to the problem, but these shortcuts are applied in situations where the degree of difference is considered acceptable.
- In some cases, multiplying two irrational numbers will produce a rational number, as in $\sqrt{2} x \sqrt{2} = 2$ but this is not true of multiplying all irrational numbers, and anytime you multiply a rational number by an irrational number, the result will always be irrational, as in $5\sqrt{3}$ because the decimal value of an irrational number never terminates, there is no integer multiplier that will resolve the irrational number to an integer value.

Online Resources related to standard N-RN.3

- Properties of rational numbers versus irrational numbers

  http://www.ck12.org/algebra/Properties-of-Rational-Numbers-versus-Irrational-Numbers/lesson/Properties-of-Rational-Numbers-versus-Irrational-Numbers/
- Irrational numbers: An explanation of the concept

  http://www.ck12.org/algebra/Properties-of-Rational-Numbers-versus-Irrational-Numbers/lecture/Irrational-Numbers:-An-Explanation-of-the-Concept/
- Square roots and irrational numbers

  http://www.ck12.org/algebra/Square-Roots-and-Irrational-Numbers
- Approximate solutions to problems involving irrational numbers

  http://www.ck12.org/algebra/Irrational-Square-Roots/lesson/Approximate-Solutions-to-Equations-

Involving-Irrational-Numbers/

- Khan Academy: Number and Quantity: The Real Number System

  https://www.khanacademy.org/commoncore/grade-HSN-N-RN

  At the bottom of the Khan Academy page is the quiz: *Recognizing rational and irrational expressions*.

Seeing Structure in Expressions

Interpret the structure of expressions

A-SSE.1: Interpret expressions that represent a quantity in terms of its context

a) Interpret parts of an expression, such as terms, factors and coefficients

- A mathematic expression is a statement of an idea. In the expression 2a + 3b, you are expressing the idea of the value of *2 times the variable a + 3 times the variable b*.
- An expression consists of *terms* and *operators*. The terms of an expression are the numbers and, or variables used to represent values: In *2a + 3b*, the first term is *2a*, the second term is *3b.*
- Mathematical operators are the symbols for addition, subtraction, multiplication and division. In *2a + 3b*, the mathematical operator is the plus symbol.
- Coefficients are the numerals that precede the variables in each of the terms. The coefficient of *2a* is *2*, and the coefficient of *3b* is *3*. When a variable is not preceded by a numeric coefficient, the value of the coefficient is considered to be one. In the equation *x + 2y*, the coefficient of *x* is *1*, and the coefficient of *y* is *2*. A coefficient may be any integer, decimal or fractional value.
- A factor is an element of value that contributes to the value of the expression. Sometimes, common factors may be eliminated to simplify an expression. For example, if you were asked to find the value of $\frac{2ax}{3bx}$ you would begin by eliminating the x that's a common factor of both *2ax* and *3bx,* to give you the value $\frac{2a}{3b}$. Another way to illustrate this is to use a simple multiplication problem:
- Find the value of $\frac{3x}{4x}$ when $x = 25$, which becomes $\frac{(3\,x\,25)}{(4\,x\,25)} = \frac{75}{100}$ but in this problem, the value

of x is irrelevant to the solution, as we have a common factor above and below the bar, so the equation will equal $\frac{3}{4}$ regardless of the value of x.

- Another example is when factoring an equation: $\frac{(a+b)(c-d)}{(a+b)(c+d)}$ once again we have a common factor: *(a+b)*, we eliminate that common factor, and get $\frac{(c-d)}{(c+d)}$.

Online Resources related to this standard

- Single variable expressions
  http://www.ck12.org/ccss/high-school:-algebra/seeing-structure-in-expressions
- Expressions with one or more variables
  http://www.ck12.org/algebra/Expressions-with-One-or-More-Variables/lesson/Expressions-with-One-or-More-Variables-Intermediate/
- Variable and expressions
  https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-writing-expressions/v/expression-terms--factors-and-coefficients

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b) Interpret complicated expressions by viewing one or more of their parts as a single entity

Sometimes you can simplify expressions by factoring out common elements. In the expression $\frac{a^2+4a-21}{a^2-9a+18}$: facator $a^2+4a-21$ by figuring out what *(a+?)(a+?)* is - *remember that subtraction is considered the same as adding a negative number*. Look for two values that when added together equal *+4*, and when multiplied equal *-21*. The values *-3* and *+7* meet those criteria, so you test by multiplying *(a-3)(a+7)* and get $a^2+4a-21$, which means you've found your numerator. Following the same steps for your denominator gives you *(a-6)(a-3), so your equation becomes* $\frac{(a+7)(a-3)}{(a-6)(a-3)}$ *,* you eliminate the *(a-3)* from above and below the line to get the value $\frac{(a+7)}{(a-6)}$. .

Online Resources related to this standard

- Simplifying rational expressions

  http://www.ck12.org/algebra/Simplifying-Rational-Expressions
- Multiplication of rational expressions

  http://www.ck12.org/algebra/Multiplication-of-Rational-Expressions
- Simplifying rational expressions
- https://www.khanacademy.org/math/algebra2/polynomial_and_rational/simplifying-rational-expressions/v/simplifying-rational-expressions-1

A-SSE.2: Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$ Write expressions in equivalent forms to solve problems

This standard involves understanding of the Distributive Property, Factoring polynomials and the properties of exponents. The Distributive Property is typically illustrated as *a(b + c) = ab + ac* and it means that when you are multiplying values contained in brackets by an external value, that external value is distributed across all of the elements within the brackets. One method of applying the distributive method to a pair of binomials is the *FOIL* method. *FOIL* stands for *First, Outside, Inside Last*. As an example, we'll multiply (x – 2)(x + 1)

First: $(\underline{x} - 2)(\underline{x} + 1) = x^2$

Outside: $(\underline{x} - 2)(x + \underline{1}) = x$

Inside: $(x \underline{- 2})(\underline{x} + 1) = -2x$

Last: $(x \underline{- 2})(x + \underline{1}) = -2$

When we assemble all of those values as one polynomial we get $x^2 - 2x + x - 2$, and when we combine all the like terms it becomes: $x^2 - x - 2$.

Factoring polynomials involves finding values that are factors of a particular value. Factoring is the reverse of multiplication, and is largely a product of pattern recognition. In order to master factoring of polynomials, you must first learn the following patterns.

$(x + y)(x + y) = x^2 + 2xy + y^2$

$(x + y)(x - y) = x^2 – y^2$

$(x – y)(x – y) = x^2 -2xy + y^2$

$(x + a)(x + b) = x^2 + x(a + b) + ab$ – This example may seem confusing to some students, so we'll do an example with integers: *(x + 1)(x + 2)*, applying FOIL gives us $x^2 + 2x + x + 2$, which combines to give us $x^2 + 3x + 2$.

Online resources related to this standard

- A-SSE.2: Seeing Structure in Expressions
  http://www.ck12.org/ccss/high-school:-algebra/seeing-structure-in-expressions
- Factoring and the Distributive Property
  https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/Factoring-simple-expressions/v/factoring-and-the-distributive-property
- Factoring Quadratic Expressions
  https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/factoring-quadratic-expressions/v/factoring-quadratic-expressions

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A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a) Factor a quadratic expression to reveal the zeros of the function it defines

A quadratic equation is defined as an expression with a single variable of degree 2, which means you're describing an equation where one of the values is squared. When one of the values in an equation is squared, it means the graph of that expression will be a curve, rather than a straight line, and you will have up to two x intercepts, . When you factor a quadratic equation into two polynomials, the values within those polynomials reveal the locations of the *x-intercepts* of your graph.

For an example, if you start with the equation $x^2 + 2x - 3$, and factor that so you get the pair of binomials (x + 3)(x – 1), then you can immediately find your x-intercepts, because if you've factored correctly, then (x + 3) and (x – 1) will both equal 0, so your x-intercepts are found by solving $x + 3 = 0$ and $x - 1 = 0$; which leads you to *x = -3 and 1*.

Online resources related to this standard

- Solving quadratics by factoring (ignore the triangle and box videos)

  https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/v/Example%201:%20Solving%20a%20quadratic%20equation%20by%20factoring

b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Completing the square is a method for finding the x-intercepts of a quadratic equation, when it's not possible to factor out integer values.

- Begin with an equation that doesn't factor easily: $x^2 + 6x - 3 = 0$
- Move the final value to the other side of the equation: $x^2 + 6x = 3$
- Find the value you must add to both sides of the equation, so the left side of the equation becomes a perfect square: $x^2 + 6x + 9 = 3 + 9$
- Which becomes: $(x + 3)^2 = 12$
- And then: $x + 3 = \pm\sqrt{12}$ which reduces to $x = \pm 2\sqrt{3} - 3$ which approximates to:

  $x = \pm 2\,(1.732) - 3$ or x = 3.464 – 3 = 0.464 and x = -3.464 – 3 = -6.464

Finding the maximum or minimum of an equation.

- The equation for finding the y-value of a quadratic equation is $\dfrac{c - b^2}{4a}$

- Remember that a quadratic equation is described by the form: ax^2+bx+c
- As an example, we'll take the values from the equation $x^2+6x-3=0$, were *a = 1*, *b = 6* and *c = -3*
- Applying this to our equation above we get:

$$\frac{(-3)-b^2}{4(1)} \text{ which becomes } \frac{-3-(-6)^2}{4,} \text{ which becomes equals} -\frac{33}{4} \text{ which equals} -8.25$$

If a quadratic equation has two x-intercepts, and the vertex of the parabola is below the x-axis (it has a negative y-value) then the parabola opens upward, and you are finding a minimum. If a quadratic equation has two x-intercepts, and the vertex is above the x-axis (it has a positive y-value) then the parabola opens downward, and you are finding a maximum value. In the case of $x^2 + 6x - 3 = 0$, , we have the negative y-coordinate, so we know that -8.25 is the minimum value for this equation.

Online resources related to this standard

- Completing the square
  http://www.ck12.org/algebra/Completing-the-Square/#all

- Completing the square
  https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/v/solving-quadratic-equations-by-completing-the-square

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c) Use the properties of exponents to transform expressions for exponential functions.

Properties of Exponents

Zero Power Property: $a^0=1$ this holds true for any value of *a*.

Division of Exponents: $\frac{a^m}{a^n}=a^{m-n}$

Multiplication of Exponents: $a^m\,x\,a^n=a^{m+n}$

Binomial to a Power: $(a+b)^m=a^m\,x\,b^m$

Roots of Powers: $\sqrt[n]{a^m}=a^{m-n}$

As an example of using these properties - find the value of x: $\sqrt[5]{a^n}+\frac{a^n}{a^5}=x$

- If we apply the Roots of Powers property, we get: $a^{n-5} + \frac{a^n}{a^5}$
- By applying the division of exponents to the right side of the equation we get $a^{n-5} + a^{n-5} = x$
- Which becomes $2a^{n-5} = x$
- Properties of exponents (scroll down to the videos labeled "HSA.SSE.B.3.C")

  http://www.ck12.org/ccss/high-school:-algebra/seeing-structure-in-expressions
- Properties of exponents

  https://www.khanacademy.org/math/algebra/exponent-equations/exponent-properties-algebra/v/exponent-properties-1

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials

A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction and multiplication; add, subtract and multiply polynomials.

This standard means that you must learn to follow mathematical conventions regarding the handling of polynomials. express the answer to a problem using only the terms that are included in the problem. When operating with integers, you normally answer with an integer value. If someone asks you "*What does 1 + 1 equal*", you would answer "2". When dealing with integers, your set of terms is the set of all integers, and you observe the conventions of integer values. Likewise, if you were asked to find the value of x in the problem $\frac{x+y}{3}=1$, you would be expected to work through the steps according to typical algebraic conventions.

Here is how you would solve our example problem following proper algebraic conventions: $\frac{x+y}{3}=1$ *we multiply by 3 to get* $x+y=3$ *then subtract y from both sides to get* x = 3 – y. In this problem we never defined the value of *y*, but we don't have to, and even if we wanted to the problem does not provide enough information to relate *y* to an integer value. As the problem only asks us to find the value of x, we found the value of x in terms of the other values included within the problem.

This standard also refers to memorizing the FOIL properties, properties of integers and prevailing patterns which apply to the multiplication of polynomials that were addressed in prior standards.

- Arithmetic with polynomials and rational expressions: HSA.APR.A.1
  http://www.ck12.org/ccss/high-school:-algebra/arithmetic-with-polynomials-and-rational-expressions
- Algebra: Arithmetic with polynomials and rational expressions: HSA.APR.A.1
  https://www.khanacademy.org/commoncore/grade-HSA-A-APR

Understand the relationship between zeros and factors of polynomials

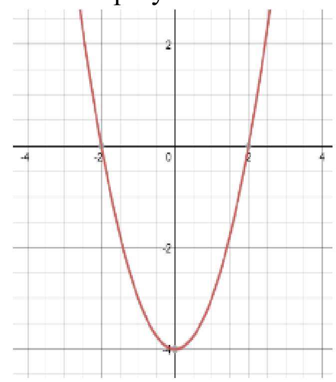
A-APR.3**:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A polynomial equation like y = x^2 -4, describes a mathematical relationship between coordinates. The table below illustrates this relationship:

x	x^2 - 4	y
-2	4 – 4	0
-1	1 – 4	-3
0	0 – 4	-4
+1	1 - 4	-3
+2	4 - 4	0

When a quadratic equation is properly factored the polynomial factors will reveal the x-coordinates where the graph crosses the x-axis. For example, with the equation $x^2 - 4$, you would have the factors (x + 2)(x – 2). You can find the x-coordinates where the curve of $x^2 – 4$ crosses the x-axis by setting each of the polynomial factors to equal zero.

- *x + 2 = 0 implies x = -2*
- *x – 2 = 0 implies x = +2*



IN this example, -4 is the minimum, because the graph opens upward.

All points along the x-axis have a y-value of zero. So the coordinates where the curve of $x^2 – 4$ crosses the y-axis are (-2,0) and (2,0). We can use these two coordinates to calculate the axis of the curve. The axis of a curve is also known as the *minimum*, if the curve opens upward, and the *maximum* if the curve opens downward. We find the x-coordinate of the axis by adding the x-coordinates for each of the two points where the curve intersects the x-axis, and dividing that answer by two.

- $\frac{x'+x''}{2}$ or, for the curve $x^2-4=0$: $\frac{-2+(+2)}{2}=0$, this tells us that the x-coordinate of the vertex of the graph equals zero, and we can find the y coordinate by substituting that zero value into our original equation: $0^2 – 4$ = our y coordinate; which means our vertex is located at the point (0, -4). Now that we know the location of the vertex of the graph, along with the two points where the curve crosses the x-axis, you can create a rough sketch of what the graph will look like.

Online resources related to this standard

- CK12.org: HSA-APR-B.3
  http://www.ck12.org/ccss/high-school:-algebra/arithmetic-with-polynomials-and-rational-expressions
- Graphing quadratics (You have to scroll down a good ways, it's a long page)
  https://www.khanacademy.org/math/algebra/quadratics

Creating Equations

Create Equations that describe numbers or relationships

A-CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include* *equations arising from linear and quadratic functions, and simple rational and exponential functions*.

There are four different means of presenting a mathematical relationship: a chart or graph, a table of values, a mathematical expression or a verbal description. This standard refers specifically to being able to draw information out of a word problem, and translate that information into one of the three other presentations.

Michael is laying a brick patio that is 8' x 12'. He is using paving stones that are 8" square. If a short-pallet of paving stones has 48 stones, and costs $117.00, and individual paving stones cost $2.72 each, and Michael likes to have an extra half-dozen stones on each job to account for accidental breakage and chipping, then how much will Michael need to budget to buy stones for this job?

- Begin by translating the dimensions to inches, because the pavers are measured in inches. 8' x 12' = 96" x 144", $\frac{96}{8}=12$ and $\frac{144}{8}=18$ so we know the area will be 12 stones wide by 18 stones long.
- 12 stones x 18 stones = 218 stones.
- $\frac{218\,stones}{48\,stones\ per\ pallet}=4\ pallets\ plus\ 26\,stones$
- Since Michael likes to have an extra 6 stones, it becomes 4 pallets plus 32 stones
- Now we calculate which is cheaper, buying 4 pallets plus 32 stones, or buying 5 pallets.
- 4 ($117.00) + 32 ($2.72) = $468.00 + $87.04 = 555.04 and 5 ($117.00) = $585, which means it's cheaper for Michael to buy the loose stones, rather than five pallets.

In the above problem we've taken a verbal description of a situation that an average working person is likely to encounter. We translated the problem into a mathematic equation, and found the solution to the problem. A contractor who just makes a rough guess on problems like these, based on a gut feeling or past experience may be accurate most of the time, but will inevitably find that on some jobs he's over or under-bought materials, and ended up losing time and money as a result.

Online resources related to this standard

- Creating equations: HSA.CED.A.1

http://www.ck12.org/ccss/high-school:-algebra/creating-equations

- Creating equations: HSA.CED.A.1

  https://www.khanacademy.org/commoncore/grade-HSA-A-CED

A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

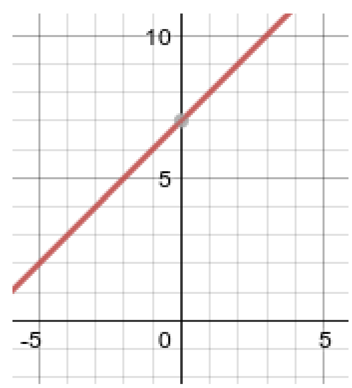
This standard involves interpreting information from word problems, creating a table of values, and graphing those values on a coordinate plane.

A fully loaded barge in still water moves at an average speed of 7 miles per hour. When moving in water where there is a current, the rate of travel is $7 + c$. When the boat is moving with the current, so the water is pushing the boat along, c is a positive number. When the boat is moving against the current, then c is negative. Graph the current on the x-axis.

- Create a table of values. Moving with the current will be represented as a positive number, and against the current is a negative value.

Current	-4	-2	0	+2	+4
Speed	3	5	7	9	11

Since the problem tells us to graph the current on the x-axis, we'll look at the figures in the above table as our x and y coordinates. Looking at the rate of travel that's given "7 + c", and knowing that we're graphing speed on the y-axis, we can now say that $y = 7 + c$. Since we're graphing the current "c" on the x-axis, we can translate that to the equation $y = x + 7$. If we graph out these points, and fit a line, we end up with the graph at the right.



$y = x + 7$

Online resources related to this standard

- HSA-CED.A.2: Graphs of linear systems

  http://www.ck12.org/ccss/high-school:-algebra/creating-equations

- HSA-CED.A.2

  https://www.khanacademy.org/commoncore/grade-HSA-A-CED

A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

This standard refers to practical limits on equations or inequalities.

- It's 146 miles from the Regional Transportation Center in Syracuse to the Albany Bus Depot. A bus company calculates a scheduled trip time of 200 ± 12 minutes. Assuming the bus leaves on time, calculate the average speed range -in mph- the driver must achieve to remain on schedule.

- First we'll take 200 minutes ± 12 minutes, which is 188 minutes, to 212 minutes.
- $\frac{188}{60}=3.13\,hours$ and $\frac{212}{60}=3.53\,hours$
- $\frac{146\,miles}{3.13\,hours}=46.64\,mph$ and $\frac{146\,miles}{3.53\,hours}=41.35\,mph$
- So the driver must maintain an average travel speed of 41.35 to 46.64 mph to remain on schedule.

As you can see, the driver is expected to remain within a given schedule, so we're limiting the problem accordingly. We aren't concerning ourselves with how long it will take if the bus only travels at 2 miles per hour, or with how much faster the trip would be if the bus drove at 292 mph. The expectations of the company are constraints on the problem. Constraints are also common with problems that have a radical on one side of the equation, but where the problem only asks you to find the positive or negative solution.

Online resources related to this standard

- HSA.CED.A.3
  http://www.ck12.org/ccss/high-school:-algebra/creating-equations
- Modeling constraints with two-variable inequalities
  https://www.khanacademy.org/commoncore/grade-HSA-A-CED

A-CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*

This principle requires you to develop facility with manipulating equations. We'll begin withhttp://www.ck12.org/ccss/high-school:-algebra/creating-equations the example provided: *V=IR*. Let's manipulate the equation to solve for the values of I and R.

- Solving for *I*: V = IR > $\frac{V}{R}=\frac{IR}{R}$ cancel out the extra *Rs* and > $I=\frac{V}{R}$
- Solving for R: $\frac{V}{I}=\frac{IR}{I}$ cancel out the extra *Is* and > $R=\frac{V}{I}$

This skill becomes especially important in geometry, where you'll often be given formulas to calculate the surface area or volume of a figure, and you'll have to plug in known data so you can calculate an unknown quantity.

- A custom aquarium fabricator needs to create a cylindrical tank that is 36" tall and has a volume of approximately 40 gallons (1 gallon = 231 in^3). What should the diameter of the tank be, to the nearest 1/2"?
- The equation for finding the volume of a cylinder is $V = \pi r^2 h$, where V = volume, π is an irrational number used in finding the area of a circle, *r* is the radius (½ the diameter) and *h* is the height of the cylinder. Since we're calculating to the nearest half inch, we'll round π to 3.14.

- $V = \pi r^2 h$: (40 x 231) = $(3.14)(r^2)(36)$
- $\frac{V}{\pi h}=r^2$: $\frac{9240}{113.1}=81.7$ dividing both sides of the equation by πh gives us the value for r^2.
- Using our calculator we find that: $\sqrt{81.7}=9.03$, the radius is ½ the diameter, so the answer is 18.06, and rounding to the nearest 1/2" gives us an answer of 18"

Online resources related to this standard:

- CK12.org: HSA-CED-A.4: Formulas for problem solving
  http://www.ck12.org/ccss/high-school:-algebra/creating-equations
- Khan Academy: HSA-CED-A.4: Manipulating formulas
  https://www.khanacademy.org/commoncore/grade-HSA-A-CED

Reasoning with Equations & Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

This standard involves showing your work, and being able to explain the steps you took in solving an equation. You've seen many examples in textbooks where a problem is solved in a stepwise fashion, and brief snippets of text appear alongside the problem, describing how the solution was reached.

- A right-circular cone has a volume of $56.6\ cm^3$ and a diameter of 6 cm. What is the height of the cone? Use $\frac{22}{7}$ to approximate the value of π.
- By looking at a reference table we see that the equation for calculating the volume of a right circular cone is $V = \pi r^2 \frac{h}{3}$.
- Since the diameter equals 6cm, and the radius is ½ the diameter, that means r = 3
- First we'll multiply both sides of the equation by 3, to get $3V = \pi r^2 h$.
- That becomes $3\,x\,56.6 = \frac{22}{7}\,x\,9\,x\,h$
- Multiplying both sides by 7 gives us $3\,x\,7\,x\,56.6 = 22\,x\,9\,x\,h$
- Dividing both sides by 22 x 9 becomes $\frac{21\,x\,56.6}{198} = h$
- 21 x 56.6 = 1188.6, so $\frac{1188.6}{198} = 6.003...$ so (because of significant figures) we round our answer to 6 cm.

One of the great textbook authors of the early Twentieth Century was Emerson White. His book *A School Algebra* was one of the all time best at providing explanations of how to solve different types of problems. Look at the explanatory sections of White's textbook -especially Chapter 1 through 5- for numerous examples of how to explain your steps towards achieving an answer to a problem.

- Emerson White: *A School Algebra*
  https://archive.org/details/aschoolalgebrad00whitgoog

A-REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

"Equations in one variable" means you are only trying to find one value in the equation.

- 3x = 48 is an equation in one variable, because you're only trying to find the value of x.
- A rational equation is one that includes a fraction (remember that a fraction is a type of ratio)
  $\frac{4x}{5}=8$ is an example of a radical equation.
- A radical equation is one that includes a root sign.
  $3\sqrt{x}=48$ is an example of a radical equation.

Solving a radical equation involves finding a common denominator – when necessary, isolating the variable on one side of the equation, and then solving for the variable.

- $\frac{8x}{4}+\frac{11x}{3}=17$
- First we multiply the the two sides of the equation by "1" to find our common denominators:
  $\frac{3(8x)}{3(4)}+\frac{4(11x)}{4(3)}=17$ which becomes: $\frac{24x}{12}+\frac{44x}{12}=17$ which equals $\frac{68x}{12}=17$
- Multiply by 12 to get rid of the denominator and we get 68x = 12(17) or 68x = 204
- $204 \div 68=3$ which means x = 3

The steps to solving a radical equation are:

1. Isolate the radical
2. Raise both sides to whatever power is needed to eliminate the radical
3. Solve the equation
4. Check for extraneous roots

- $\frac{3\sqrt{x}}{42}=\frac{1}{2}$ first we want to isolate the radical so we multiply both sides by *42* to get
  $3\sqrt{x}=\frac{42}{2}\, which\, becomes\, 3\sqrt{x}=21$

- Divide both sides by 3 to get $\sqrt{x}=7$ and square both sides to get $x = 49$.

Extraneous solutions may occur when you are working with radical equations, especially when dealing with equations where you have a quadratic equation on one side.

- $x+4=\sqrt{x+10}$ which becomes $(x + 4)^2 = x + 10$
- which becomes $x^2 + 8x + 16 = x + 10$
- which reduces to $x^2 + 7x + 6 = 0$ which factors to $(x + 1)(x + 7)$
- which implies that x = -1 or -7
- When you check your answers, you see -7 does not work because $-7+4\neq\sqrt{-7+10}$ this means that -7 is an extraneous solution, because it does not satisfy the original equation.

Online Resources related to this standard

- HSA.REI.A.2: Reasoning with equations and inequalities
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy: Working with radical equations
  https://www.khanacademy.org/math/algebra/exponent-equations/radical_equations/v/solving-radical-equations-1

A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

An equation describes a set of values that may be graphed on a line. Any point along that line is part of the solution set for that equation. An equation is typically identified by the presence of an *equals* sign. An inequality describes a range of values. The equation for an inequality will typically include the greater than or less than signs.

“ > “ is the *greater than* sign and “ < “ is the *less than* sign

When you see an extra line below the greater than or less than sign, it means it is a “greater than or equal to” or a “less than or equal to” symbol.

$a\leqslant b$ means *a is less than or equal to b*

$a\geqslant b$ means *a is greater than or equal to b*

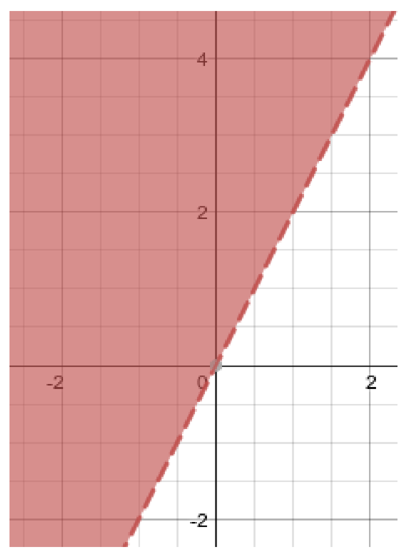
One trick for remembering which is which, is to think of the *greater than* and *less than* signs as alligator mouths reaching for the bigger bite. On a graph, an equation is indicated by a line, while an inequality is represented by a shaded region. The shaded region represents all of the values on the graph that satisfy the mathematical relationship described in the inequality. The first step to graphing an equation is to create a table of values for your horizontal and vertical axes. The standard convention for algebra students is to refer to the values along your horizontal axis as your x-values, and the values on your vertical axis as your y-values. On your table of values, select the values of x that you're going to graph.

y = x + 3					
x -value	-2	-1	0	1	2
x + 3 equals	1	2	3	4	5

When you begin a problem by selecting a range of values for x that you're going to graph, it means you're selecting x as your *independent variable*. Your equation or inequality describes a relationship between the values of x and y, and the value of y will *depend* upon which values of x you choose to graph, which means that y is your *dependent variable*. In the table above, the values in the *x + 3 equals* row are your y-values. To create your graph for this equation, begin by placing dots at each set of coordinates in the table: (-2,1), (-1,2), (0,3), (1,4), (2,5); then draw to connect all of those dots. You've now graphed an equation.

To graph an inequality you solve your equation as if the *greater than* or *less than* sign were an equals sign, but instead of drawing a single line, you create a dotted or broken line, and shade above or below that line, according to whether you are looking for the range of values that are greater than or less than the value described in the inequality. In the example shown on the previous page, when x = -1, all of the values for y that are greater than -2 are shaded in, because all of those values are part of the solution set for that inequality, at that particular value of x.

There are a variety of problems where an exponent is represented by a letter. These problems are often referred to as *Nth power problems*, and involve a value that increases or decreases over a cycle of time. The equation for calculating compound interest is one example of an Nth power problem. The equation for finding compound interest is $P(1 + i)^n$, where *P* is your principle – the amount that is originally invested; *i* is your interest rate - expressed as a decimal value; and *n* is number of time cycles that you allow the investment to appreciate. Most of these problems will refer to an annual interest rate, and *n* refers to a number of years. To find the interest on $10,000 that is invested at an annual interest rate of 8% over a period of 3 years you would multiply: $10,000 $(1.08)^3$ which equals $10,000 (1.2957) or $12, 957.



Notice how in this graph of the inequality y > 2x, the entire region of the graph where y > 2x is shaded.

Online resources related to this standard

CK-12.org: HSA.REI.B.3 – review all of the modules listed beneath the standard heading

http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities

Khan Academy: HSA.REI.B.3 – scroll down the page to find the listing, then review all of the modules listed beneath the standard heading.

http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities

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A-REI.4a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x\text{-}p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Completing the square is a method for finding the solutions to equations that do not factor easily.

- $x^2 - 4x - 3 = 0$ Completing the square begins with a problem whose values do not readily lend themselves to inspection or factoring.
- $x^2 - 4x = 3$ The first step is to move the final value to the other side of the equation
- $x^2 - 4x + 4 = 7$ Where'd the "4" come from? We add 4 to each side of the equation, because it allows us to create an equation on the left that is an easily factored value: $(x - 2)^2 = x^2 - 4x + 4$. To complete the square you add a value to both sides of the equation so the value on the left is transformed into a square of $(x + y)$ or $(x - y)$.
- $(x-2)^2 = 7$ becomes $x - 2 = \sqrt{7}$
- Add 2 to both sides to get $x = \sqrt{7} + 2$ for these types of problems you must consider both the positive and negative roots of 7. Using only the radical sign suggests that you're referring only to the primary -or "positive" root- so you should write your answer as: $x = \pm\sqrt{7} + 2$ to make it clear that you're referring to both the positive and negative square roots of seven.

Online resources related to this standard

CK-12.org: HSA.REI.B.4a – review all of the modules listed beneath the standard heading

http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities

Khan Academy: HSA.REI.B.4a – scroll down the page to find the listing, then review all of the modules listed beneath the standard heading.

http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities

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A-REI.4b: Solve quadratic equations by inspection *(e.g., for x2=49)*, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

This standard is essentially a wrap-up on how to handle quadratic equations. Here we will review the various methods for handling quadratic equations.

Solving by inspection means recognizing a square root.

- $x^2 - 3 = 0$ becomes $x^2 = 3$ or $x = \pm\sqrt{3}$
- Keep in mind that when you're dealing with quadratic equations, you need both the positive and negative square roots.

Factoring means recognizing the values used using the addition and multiplication method, and checking your work using FOIL.

- $x^2 - 2x = 15$
- $x^2 - 2x - 15 = 0$ Now we find two values that when added together = -2, and when multiplied together equal -15. 3 and -5 should work
- $(x+3)(x-5)$ apply FOIL and get: $x^2 - 5x + 3x - 15$ which equals $x^2 - 2x - 15$

Completing the square was referred to in standard 4a.

- **The Quadratic Formula** is a method for finding solutions to quadratic equations that do not lend themselves to any of the other solution methods. The quadratic formula is $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$
- remember that standard form for a quadratic equation is $ax^2 + bx + c$
- For the equation $x^2 + 5x - 3$, we translate the values from the equation so a = 1, b = 5 and c = -3
- Inspection does not work for this problem. Factoring does not provide an easy solution. Completing the square would require working with the fraction 5/2, so in this case it's easiest to use the quadratic formula. It is often easier to use the quadratic formula than it is to deal with completing the square when fractions are involved.
- so $x = \frac{-5 \pm \sqrt{[5^2 - 4(1)(-3)]}}{2(5)}$ or $x = \frac{-5 \pm \sqrt{[25 - (-12)]}}{10}$ or $\frac{-5 \pm \sqrt{37}}{10}$
- since the square root of 37 = 6.08, we get $\frac{-5 \pm 6.08}{10}$ equals 0.1 or -1.1.

Online resources related to this standard

- CK12.org: HSA.REI.4B.A
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy: HSA.REI.B4.A
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

Solve systems of equations

A-REI.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

This standard refers to being able to manipulate the form of an equation. Later standards address being able to solve systems of equations through processes of substitution and addition. This standard is preparatory, and involves being able to put equations into a form so that you will be able to perform substitution and addition of equations.

Arranging values: this skill will allow you to arrange sets of equations into a common form so you can find solutions through the addition of equations

- Start with the pair of equations 3b = 14 - 2a and 4a = 7 + b. Change these two equations so they are both of the form $a + b = x$.

- For $3b = 14 - 2a$, simply add $2a$ to both sides of the equation to get: $2a + 3b = 14$.
- For $4a = 7 + b$, subtract b from both sides to get $4a - b = 7$.
- Now you end up with: $2a + 3b = 14$
  $4a - b = 7$ Now we'll multiply this bottom equation by 3, so we end up with +3b on top, and -3b on the bottom.
- So we get
  $$\begin{aligned} 2a + 3b &= 14 \\ \underline{12a - 3b} &\underline{= 21} \\ 14a \quad\quad &= 35 \end{aligned}$$
- Reducing that gives us $a=\frac{35}{14}$ which we divide by $\frac{7}{7}$ to get $a=\frac{5}{2}$
- $2a + 3b = 14$, which means $5 + 3b = 14$, or $3b = 9$, which reduces to $b = 3$.
- So the solution is $\left(\frac{5}{2}, 3\right)$

Online resources related to this standard:

- CK12.org: HSA.REI.5
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy: HSA.REI.B4.A
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

A-REI.6: Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

In the previous standard you solved systems of equations using the process of addition to isolate the first value in the equation, then substituted the first value into the equation to derive your second value. For this standard, you'll isolate values of x and y to determine your relatives values, then substitute those relative values into your second equation, to find the solution to the problem. Isolating values of x and y means getting equations into the form $x = y + a$ or $y = x + a$.

Isolating values of x and y: this skill will prepare you for solving problems through substitution.

Solve the following problems for x:

- $3x + y = 18$
  - Subtract y from both sides to get: $3x = 18 - y$
  - Divide both sides by 3 to get $x = 6 - \frac{y}{3}$
- $3x + 3y - 6 = 0$
  - Subtract $3y$ and add 6 to both sides to get $3x = -3y + 6$
  - Divide both sides by 3 to get $x = -y + 2$

Solving problems through substitution means yetting the problem in the form $y = x + a$ or $x = y + a$, then substituting the value of x or y into the equation.

- $3x + y = 13$
  $2y = 16 – x$
- $y = 13$; subtract 3x from both sides to get $y = 13 – 3x$

- Insert that value for y into the second equation to get $2(13 - 3x) = 16 - x$
- $26 - 6x = 16 - x$; add 6x to both sides to get $26 = 16 + 5x$
- Subtract 16 from both sides to get $10 = 5x$, which reduces to $x = 2$
- IF $y = 13 - 3x$, then when $x = 2, y = 13 - 3(2)$ or $y = 7$.

Online resources related to this standard

- CK12.org: HSA.REI.5
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy: HSA.REI.6
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

Represent and solve equations and inequalities graphically

A-REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

When you begin a problem by selecting a range of values for x that you're going to graph, it means you're selecting x as your *independent variable*. Your equation or inequality describes a relationship between the values of x and y, and the value of y that you are presenting will *depend* upon which values of x you choose to graph, which means that y is your *dependent variable*. Even though you are only graphing a tiny part of the solution set described by that mathematical relationship, the relationship between the values of x and y persists for all possible values. This relates to the definition of a line in geometry which states that "a line is a length that has no breadth, and extends on forever".

At the left is the graph of $x = y$. In this example we're only looking at the graph to four units out, but for the sake of this problem imagine that this line extends continuously. For every value of x, there is a corresponding value of y: when x = 1, y = 1; when x = 1191, y = 1191, etc. For every possible value of x that exists along that line or curve there is a corresponding value of y.

Online Resources related to this standard

- CK12.org HSA.D.REI.10: Graphs of Linear Systems and Solving Systems of Lines, Quadratics and Conics.
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy. HSA.D.REI.10: Intersecting Functions and Systems of Non-Linear Equations
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

A-REI.11: Explain why the x-coordinates of the points where the graphs of the equations *y=f(x)* and *y=g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and *g(x)* are linear, polynomial, rational, absolute value, exponential and logarithmic functions.

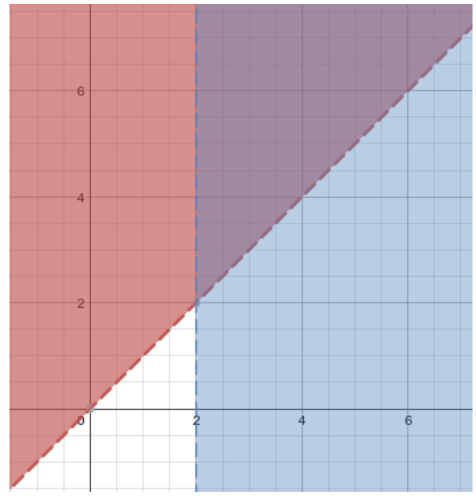
When you graph an equation beginning with the from $y = x + a$, you are setting *y* as your *dependent variable*. You are selecting the values of *x* that you're going to graph, and the values of y that you are graphing depend upon their relationship to the values for x. One way to think of independent and dependent variables is as *input* and *output*. The range of x-values that you choose to graph is what you're putting into the equation -or system of equations- and the y-values that you graph represent the output. When you are graphing two equations, the solution for those two equations is the point on the graph where the two lines intersect, or where the outputs of the two equations are equal. Because the y-values are your dependent variables, and represent the output of the system as a response to the input of x-values, it is the x-values that are ultimately the solution or solutions to the problem.

When you graph a system of two equations that are both straight lines, you will have only one solution, because two straight lines will only intersect in one location. If you are graphing a straight line and a parabola, you can have up to two points of intersection. If you graph a complex figure, or a sinusoidal curve, you may have up to an infinite number of solutions.

Online resources related to this standard

- CK12.org HSA.D.REI.11: Graphs of Linear Systems and Solving Systems of Lines, Quadratics and Conics.
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy. HSA.D.REI.11: Intersecting Functions and Systems of Non-Linear Equations
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

A-REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.



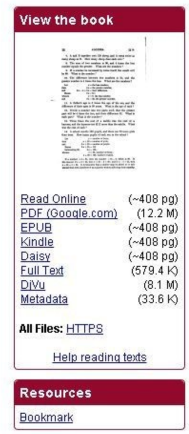
This standard is very similar to REI.11, except that the solution set for an inequality is a range of values, rather than a single point, so you're looking for two overlapping regions. In the example shown here you can see the graph of the system of inequalities $x < 2$ and $x < y$. In this example the range of values that satisfy the inequality $x < 2$ are shaded in blue, and the range of values that satisfy the equation x < y are shaded in red. The purple overlapping area is the range of values that satisfy both equations, and are therefore part of the solution set to this system of equations.

Online resources related to this standard

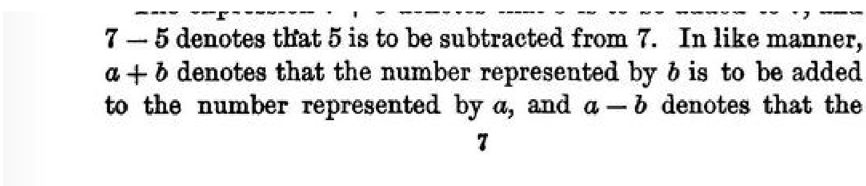
- CK12.org HSA.D.REI.12: Graphs of Linear Systems and Solving Systems of Lines, Quadratics and Conics.
  http://www.ck12.org/ccss/high-school:-algebra/reasoning-with-equations-and-inequalities
- Khan Academy. HSA.D.REI.12: Intersecting Functions and Systems of Non-Linear Equations
  https://www.khanacademy.org/commoncore/grade-HSA-A-REI

Appendix: Resources

Users Left versus Screen Left: When referring to navigation of web-pages, you'll see references to *Users Left or Right*. This is similar to Stage directions for an ac ***Read Online*** tor. Users Left refers to your left as you're facing the screen.



Archive.org textbooks: All of the Archive.org links in this booklet will refer you to the Archive.org catalog page for that title. Clicking on the *Read Online* link will open the book in your browser window. Right clicking on the PDF link will open a drop-down menu with a *Save As* option, so you can keep a copy of the book for your own use. The EPUB, Kindle, Daisy and DjVu links are for use with assorted e-reader devices. It is important to note the difference between the printed page number, and the file page number with these documents. In this handout, the page number that's given will always refer to the printed page number that appears at the bottom of the scanned page, rather than the file page number. This is because the file page number can vary depending upon the format of the file.


$7-5$ denotes that 5 is to be subtracted from 7. In like manner, $a+b$ denotes that the number represented by b is to be added to the number represented by a, and $a-b$ denotes that the

7

The printed page number appears at the bottom of the scanned page. The file page number can vary by formats. This example was taken from the *Read Online* version of the file, the printed page 7 is on file page 13.

CK12.org links

On the CK12.org pages you'll see headings for each standard, over tables that are labeled with various sub-topics within that standard. On some of the pages clicking on the sub-topic will open up a practice test. Other times you'll get print reference materials or instructional videos. In cases where multiple standards are referenced on a single page, it's best to help avoid confusion by not skipping ahead, and reviewing videos, texts and quizzes related to prior standards when you feel you need a review.

Khan Academy Videos

On the Khan Academy pages you'll a navigation bar to the Users left of the video screen. Links to instructional videos are marked by an icon that looks like a VCR play button, along with the video title. Khan Academy groups videos by subject, so all of the videos in a set are usually related to the standard being addressed. In some cases there will be a note telling you to focus on particular videos and to skip over others. On some of the pages you'll see videos that relate to more than one standard. It's recommended that you only concern yourself with the videos related to the standard you're currently working on, and go back to earlier videos if you feel you need to review those concepts.

Many of the Khan Academy links will take you to a quiz page. If you look in the lower right hand of the quiz window you'll see a link that's marked *"Stuck? Watch a Video"*, which will provide additional instruction on the topic.